Name: _____

Unit 4: Triangle Trigonometry

Objective 2.04 **Use trigonometric (sine, cosine) functions to model and solve problems; justify results.**

- Solve using tables, graphs, and algebraic properties.
- Create and identify transformations with respect to period, amplitude, and vertical and horizontal shifts.
- Develop and use the law of sines and the law of cosines.

Day	Topic	Class work
1	**7.2 Right Triangles** -trig ratios -solving right triangles -omit problems like (21-36 on p 462)	
2	**Applications of Right Triangles**	
3	**Area of a Triangle**	**Quiz 1 (Days 1-2)**
4	**7.4 Law of Sines**	
5	**7.5 Law of Cosines**	
6	Review	
7	**TEST**	**TEST**

Day 1: Things to recall from Geometry about right triangles:

- The angles of a triangle add up to 180 degrees.
- One angle is 90 degrees, therefore leaving the other angles to be acute (less than 90 degrees). Why? Because if the angles add up to 180 and one is 90, the other two have to be less than 90 together or otherwise it would be greater than 180 degrees.
- The side across from the right angle is called the hypotenuse. This is always the longest side.
- θ = theta, the measure of an angle (could be degrees or radians but we will use degrees for right now).
- If you know 2 sides of a right triangle, you can find the other because of the Pythagorean Theorem. $a^2 + b^2 = c^2$ (Whatever it equals squared must be the hypotenuse. It is NOT necessarily side c each time in each triangle, it just depends on how it is labeled)
- Each angle is represented with a capital letter and its corresponding side is represented by the lower case letter of that.

Pythagorean Theorem

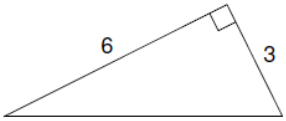
a. Pythagorean Theorem is used to find missing sides in a triangle.

b. "a" and "b" represent the _____

c. "c" represents the _____

d. Examples: Find the missing sides using Pythagorean Theorem

i.



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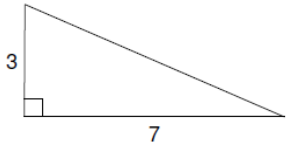

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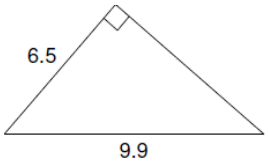

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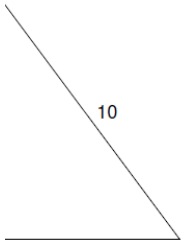
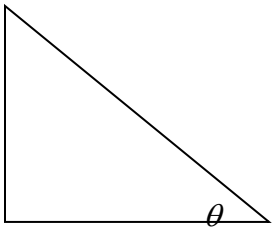
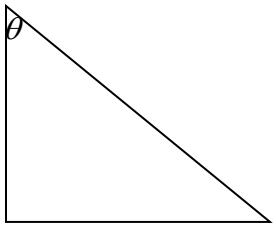


Labeling Triangles

In a triangle, depending on where theta is, depends on where the side that is opposite or adjacent is. Opposite – straight across from (opposite of it), and adjacent – right next to.
Ex:





SOHCAHTOA

SOHCAHTOA is used to help find missing sides and angles in a right triangle when Pythagorean Theorem does not work!

S (sine) **O** (opposite) **H** (hypotenuse) →

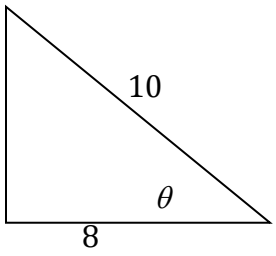
C (cosine) **A** (adjacent) **H** (hypotenuse) →

T (tangent) **O** (opposite) **A** (adjacent) →

CSC=---------- SEC= ---------- COT=----------

Given the following, find the six trig ratios. (This means, don't find angles, just set up the sides based on the ratios under SOH-CAH-TOA).

1.



2. Find all 6 trig ratios if $\sin\theta = 3/5$

3. Find all 6 trig ratios is $\cot\theta =$

Finding Missing Sides of Triangles

Setting up Trigonometry Ratios and Solving for Sides

ii. _____ (NOT the right angle)

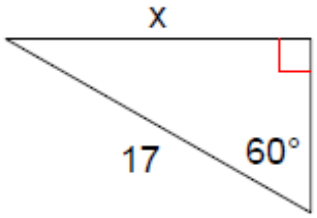
iii. _____ (Opposite, Adjacent, Hypotenuse)

iv. _____:

- ✓ _____ if we have the opposite and hypotenuse
- ✓ _____ if we have the adjacent and the hypotenuse
- ✓ _____ if we have the opposite and the adjacent

v. Set up the proportion and solve for x!

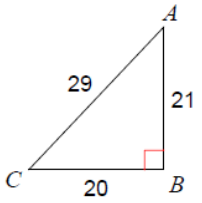
Example:



Setting up Trigonometry Ratios and Solving for Angles

i. Select a given angle (NOT the right angle)
ii. Label your sides (Opposite, Adjacent, Hypotenuse)
iii. Decide which trig function you can use:
   - ✓ SOH if we have the opposite and hypotenuse
   - ✓ CAH if we have the adjacent and the hypotenuse
   - ✓ TOA if we have the opposite and the adjacent
iv. Solve the equation … remember to you your inverses!

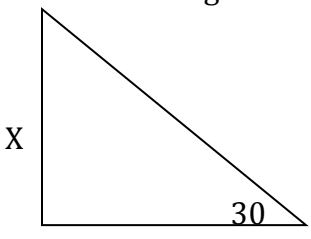
Example: Find the measure of angle A



Practice:

I. Find the missing side or angle.

1.



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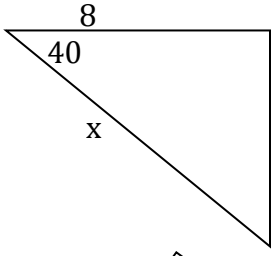

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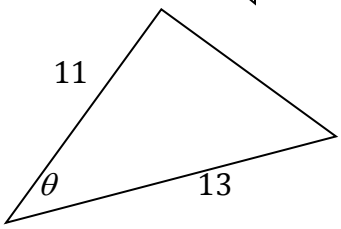
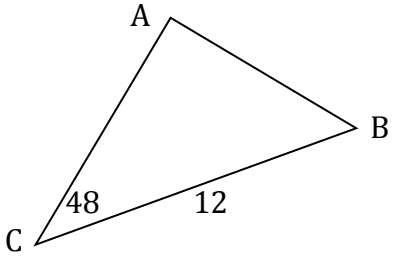


II. If it says to SOLVE the triangle, you want all 3 sides and all 3 angles.

1.



III. Find the remaining trig ratios

1. $\cos\theta = 4/\sqrt{41}$

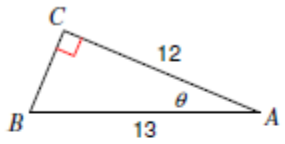
2. $\csc\theta = 41/9$

3. $\tan\theta = 5/13$

Practice:

I. Find the measure of each missing angle.

1)



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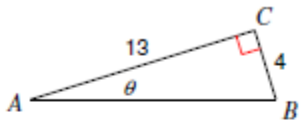

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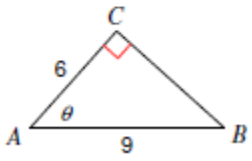

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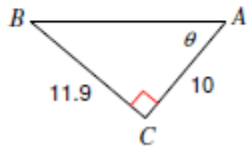

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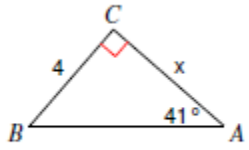

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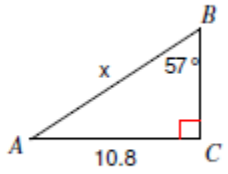

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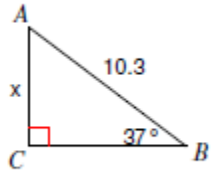

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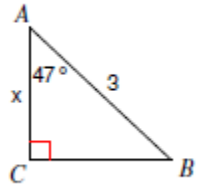
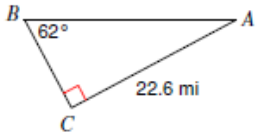


III. Solve the triangle

17)



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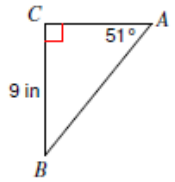

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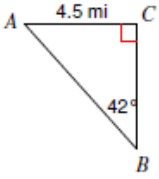

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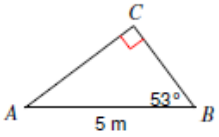
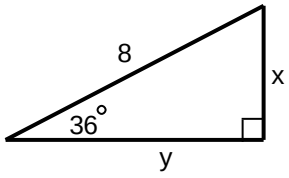
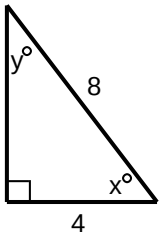
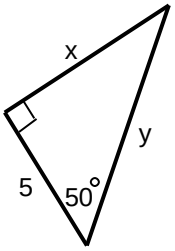
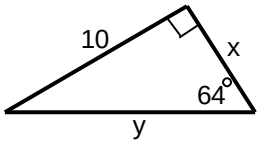
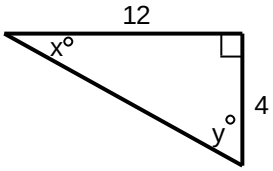
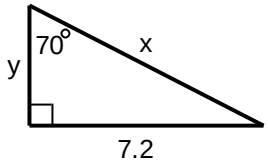
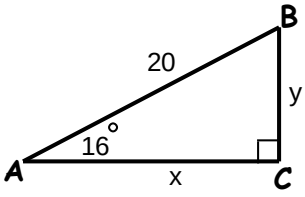
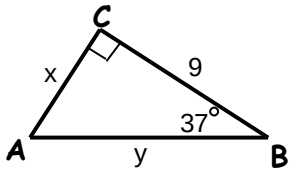


IV. Solve the following word problem.

1. A power line snaps in half because of a tornado. It breaks into 2 pieces and forms a right angle with the ground. The top of the power line rests 27 feet from the base of the pole and forms a 20^{o} angle with the ground. Find the original height of the power line before the storm.

SOHCAHTOA Sides and Angles Practice

For each of the following, write the equation to find the missing value. Then rewrite the equation that you will enter in your calculator. Round your final answer to the nearest tenth.

1. $x \approx$ _____ $y \approx$ _____ 	2. $x \approx$ _____ $y \approx$ _____ 
3. $x \approx$ _____ $y \approx$ _____ 	4. $x \approx$ _____ $y \approx$ _____ 
5. $x \approx$ _____ $y \approx$ _____ 	6. $x \approx$ _____ $y \approx$ _____ 
7. $x \approx$ _____ $y \approx$ _____ $m\angle B =$ _____ 	8. $x \approx$ _____ $y \approx$ _____ $m\angle A =$ _____ 

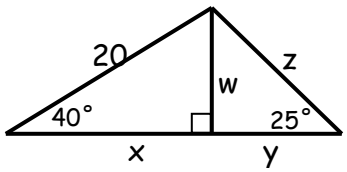
9.

w ≈ _____

x ≈ _____

y ≈ _____

z ≈ _____

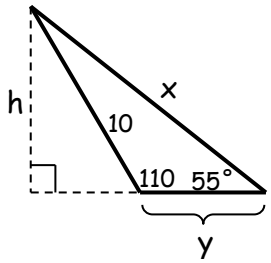


10.

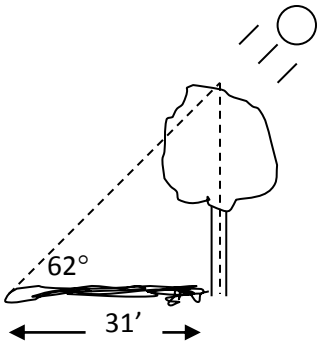
h ≈ _____

x ≈ _____

y ≈ _____



11. How tall is the tree?



12. A man who is 6 feet tall is flying a kite. The kite string is 75 feet long. If the angle that the kite string makes with the line horizontal to the ground is 35°, how far above the ground is the kite?

13. A ladder 14 feet long rests against the side of a building. The base of the ladder rests on level ground 2 feet from the side of the building. What angle does the ladder form with the ground?

14. A 24-foot ladder leaning against a building forms an 18° angle with the side of the building. How far is the base of the ladder from the base of the building?

15. A road rises 10 feet for every 400 feet along the pavement (not the horizontal). What is the measurement of the angle the road forms with the horizontal?

16. A 32-foot ladder leaning against a building touches the side of the building 26 feet above the ground. What is the measurement of the angle formed by the ladder and the ground?

17. The directions for the use of a ladder recommend that for maximum safety, the ladder should be placed against a wall at a 75° angle with the ground. If the ladder is 14 feet long, how far from the wall should the base of the ladder be placed?

Day 2: Angle of Elevation/Depression Application Problems

Angle of Elevation	Angle of Depression

1. A tree casts a 5m shadow. Find the height of the tree if the angle of elevation of the sun is 32.3º.
Sketch: **Work:** **Answer:**

2. A ladder 10.4 m long leans against a building that is 1.5 meters away. What is the angle formed by the ladder and the building?
Sketch: **Work:** **Answer:**

3. A ladder 8.6 m long makes an angle of 68º with the ground as it leans against a building. How far is the foot of the ladder from the foot of the building?
Sketch: **Work:** **Answer:**

4. The angle of depression from the top of a cliff 800 meters high to the base of a log cabin is 37º. How far is the cabin from the foot of the cliff?
Sketch: **Work:** **Answer:**

6. From a point on the ground 500 ft from the base of a building, it is observed that the angle of elevation to the top of the building is 24º and the angle of elevation to the top of a flagpole atop the building is 27º. Find the height of the building and the length of the flagpole.
Sketch: **Work:** **Answer:**

5. Mrs. Roberts stands 25ft from the flagpole. She looks and the angle of elevation to the top of the flagpole is 45 degrees. Find the height of the flagpole.
Sketch: **Work:** **Answer:**

Day 2: Trig Elevation/Depression Problems.

1.The angle of elevation of the top of a building from a point 100 feet away from the building is 22°. Find the height of the building.

2. The Sears Tower stands 1,451 feet tall. A person across the street is 30 feet away from the foot of the tower. Find the angle of elevation to the top of the tower.

3. An airplane is flying at a height of 2 miles above the ground. The distance along the ground from the airplane to an airport is 5 miles. Find the airplane's angle of elevation.

4. The angle of depression of a buoy from a point on a lighthouse 100 feet above the surface of the water is 3°. Find the distance the buoy is from the lighthouse.

5. A bird sits on top of a 15-foot lamppost. The angle of depression from the bird to the feet of an observer standing away from the lamppost is 35°. Find the distance between the bird and the observer.

6. If a plane that is cruising at an altitude of 30,000 feet wants to land at Bush Field, it must begin its descent so that the angle of depression to the airport is 7°. How far is the plane from the airport?

7. From the top of a 35 meter cliff, Lori spots a hiker at an angle of depression of 62°. Assuming Lori can sprout tentacles with which to snatch the hiker from the path (to eat the hiker, natch), how long must Lori's new demons pawn appendages be to reach the tasty morsel of a hiker?

8. Josee wanted to measure the depth of the sink hole that opened on Amelia Avenue this morning. She measured the angle of depression to the lowest point to be 35°. She also measured the distance across the sinkhole to be 38 feet. How deep is the sinkhole? As it turns out (as these things so often do), the sinkhole is really an attempt by the mole people to rise up and overthrow our way of life, in 7 sentences describe how we should best defend ourselves from the mole menace.

10. Two towers (Barad-dûr and Orthanc) face each other separated by a distance 2000m . As seen from the top of Barad-dur, the angle of depression of the second tower's base is 60° and that of the top is 30°. Based on this, how tall is Orthanc?

11. Two points on the same side of a tree are 65 feet apart. The angles of elevation of the top of the tree are 21° from one point and 16° from the other point. What kind of tree is it? If you could be a tree, what kind of tree would YOU be? Really, though, how tall is the tree?

12. A plane is 120 miles north and 85 miles east of an airport. Find its straight line distance from the airport.

13. As a hot-air balloon rises vertically, its angle of elevation from a point A, which is 110 kilometers from the point B, which is directly underneath the balloon, changes from 19°to 38°. What is the elevation of the balloon.

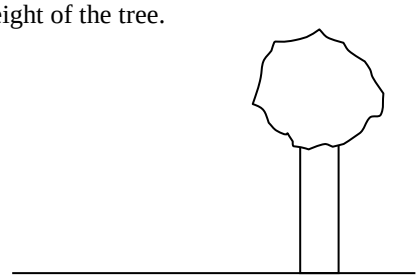
Angles of Elevation & Depression Practice

Draw a picture, write a trig ratio equation, rewrite the equation so that it is calculator ready and then solve each problem. Round measures of segments to the nearest tenth and measures of angles to the nearest degree.

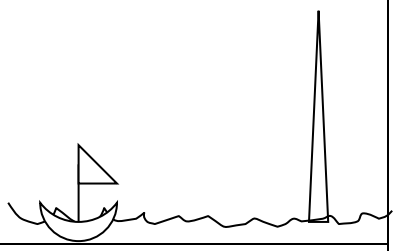
_____1. A 20-foot ladder leans against a wall so that the base of the ladder is 8 feet from the base of the building. What is the ladder's angle of elevation?

_____2. A 50-meter vertical tower is braced with a cable secured at the top of the tower and tied 30 meters from the base. What is the angle of depression from the top of the tower to the point on the ground where the cable is tied?

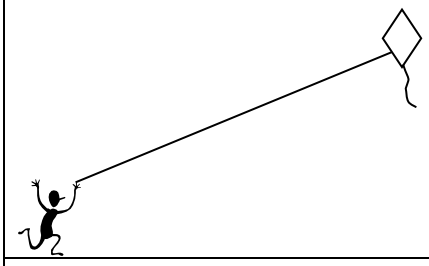
_____3. At a point on the ground 50 feet from the foot of a tree, the angle of elevation to the top of the tree is 53°. Find the height of the tree.



_____4. From the top of a lighthouse 210 feet high, the angle of depression of a boat is 27°. Find the distance from the boat to the foot of the lighthouse. The lighthouse was built at sea level.

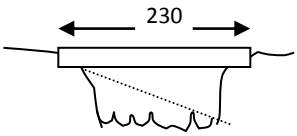


_____5. Richard is flying a kite. The kite string has an angle of elevation of 57°. If Richard is standing 100 feet from the point on the ground directly below the kite, find the length of the kite string.



_____6. An airplane rises vertically 1000 feet over a horizontal distance of 5280 feet. What is the angle of elevation of the airplane's path?

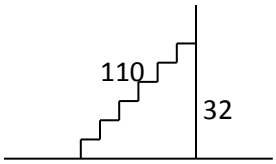
_____7. A person at one end of a 230-foot bridge spots the river's edge directly below the opposite end of the bridge and finds the angle of depression to be 57°. How far below the bridge is the river?



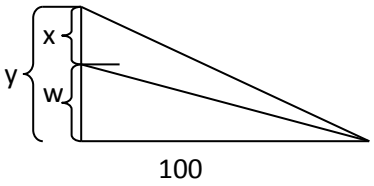
_____8. The angle of elevation from a car to a tower is 32°. The tower is 150 ft. tall. How far is the car from the tower?

_____9. A radio tower 200 ft. high casts a shadow 75 ft. long. What is the angle of elevation of the sun?

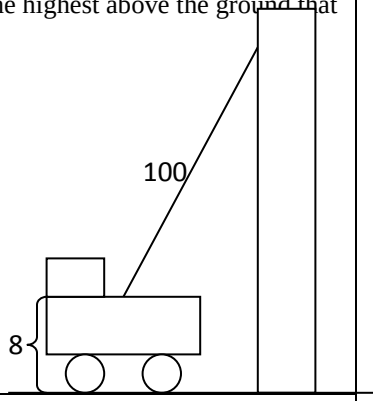
_____10. An escalator from the ground floor to the second floor of a department store is 110 ft long and rises 32 ft. vertically. What is the escalator's angle of elevation?



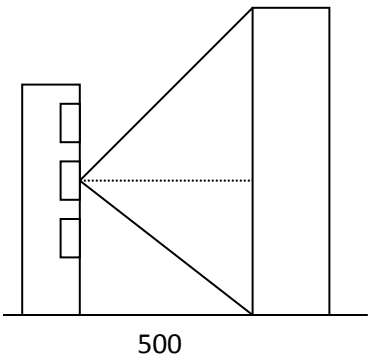
_____11. A rescue team 1000 ft. away from the base of a vertical cliff measures the angle of elevation to the top of the cliff to be 70°. A climber is stranded on a ledge. The angle of elevation from the rescue team to the ledge is 55°. How far is the stranded climber from the top of the cliff? (Hint: Find y and w using trig ratios. Then subtract w from y to find x)



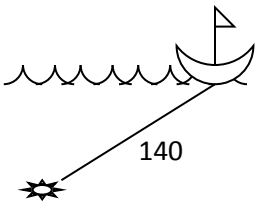
_____12. A ladder on a fire truck has its base 8 ft. above the ground. The maximum length of the ladder is 100 ft. If the ladder's greatest angle of elevation possible is 70°, what is the highest above the ground that it can reach?



_____13. A person in an apartment building sights the top and bottom of an office building 500 ft. away. The angle of elevation for the top of the office building is 23° and the angle of depression for the base of the building is 50°. How tall is the office building?



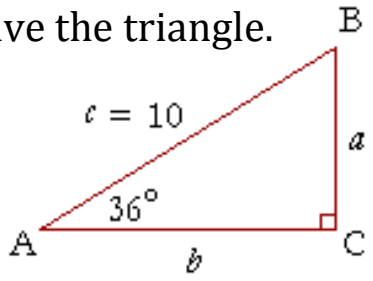
_____14. Electronic instruments on a treasure-hunting ship detect a large object on the sea floor. The angle of depression is 29°, and the instruments indicate that the direct-line distance between the ship and the object is about 1400 ft. About how far below the surface of the water is the object, and how far must the ship travel to be directly over it?



Quiz Review: Triangle Trig

1. A triangle has an acute angle such that $sin\theta = \frac{3}{8}$. Find the other five trigonometric ratios.

2. Solve the triangle.

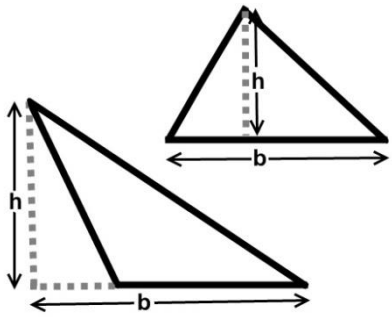


3. John is standing 20 feet from a tree. The angle of elevation to the top of the tree is 15º. There is a tree house on top of the tree and the angle of elevation to the top of the tree house is 18º. How tall is the tree house?

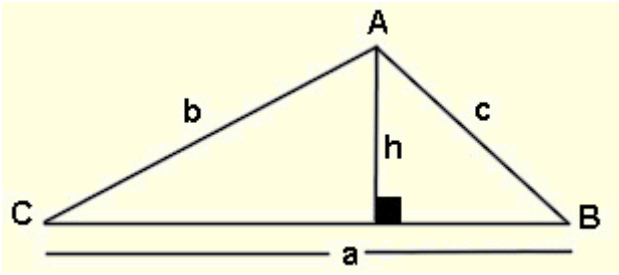
4. From the top of a barn 25 feet tall, you see a cat on the ground. The angle of depression of the cat is 40º. How many feet, to the *nearest foot,* must the cat walk to reach the barn?

Area of a Triangle

The most common formula for area of a triangle is $A = \frac{1}{2}bh$ where b is base and h is height.



But what if we don't know the height? GET CREATIVE ☺



By using the right triangle on the left side of the diagram, and our knowledge of trigonometry, we can state that:

$\sin C = \frac{h}{b}$

$bsin\ C = h$

Thus, the height can be expressed as $bsin\ C$

If we substitute this in to our common area formula we get:

$A = \frac{1}{2}absinC$

Where a and b are adjacent sides and C is the included angle (THIS HAS TO BE TRUE!)

Example 1: Find the area of a triangle with the sides of length 7 and 9 and the included angle of 72^o.

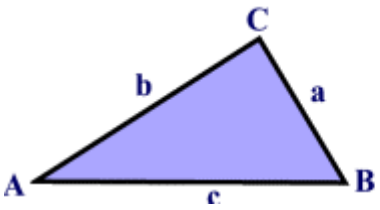
Example 2: Find the area of a triangle with the sides of length 10 and 22 and the included angle of 10^o.

Example 3: Find the area of an equilateral triangle with side length of 10.

Example 4: In ΔABC, AB = 12 meters and AC = 20 meters. If the area of the triangle is 77 sq. meters, find the measure of <A, to the nearest degree.

Example 5: A farmer has a triangular field where two sides measure 450 yards and 320 yards. The angle between these two sides measures 80º. The farmer wishes to use an insecticide that costs $4.50 per 100 sq. yards or any part of 100 yds. What will it cost to use this insecticide on this field?

Example 6: Find the area of the triangle below.

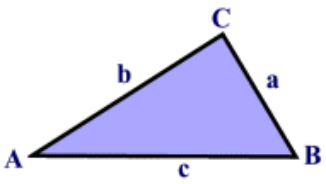


$m\angle A = 58°$

$a = 25$

$b = 22$

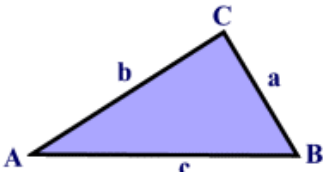
Determine how many triangles can be constructed, then solve the triangle and find its area(s).

1. 

$m\angle A = 20°$

$a = 20$

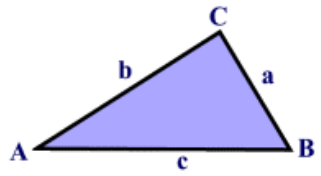
$b = 32$

2. 

$m\angle A = 65°$

$a = 18$

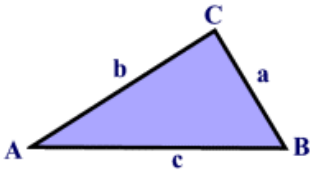
$b = 22$

3. 

$m\angle A = 20°$

$a = 8$

$b = 14$

4. 

$m\angle A = 155°$

$a = 12.5$

$b = 8.4$

Find the area of the triangle.

1. In ΔABC, $AB = 10$, $AC = 8$, and $m\angle A = 45º$. Find the area of ΔABC, to the *nearest tenth* of a square unit.

2. In an isosceles Δ, the two equal sides each measure 24 meters, and they include an angle of 30º. Find the area of the isosceles triangle, to the *nearest sq. meter.*

3. A triangle has two sides of 30 meters and 26 meters, and the angle between them is an obtuse angle. If the area of the triangle is 300 sq. meters, find the measure of the obtuse angle (to the

4. If the vertex angle of an isosceles triangle measures 30º and each leg measures 4, find the area of the triangle.

5. The vertex angle of isosceles triangle ABC measures 30º, and each leg has length 20. What is the area of ABC?

6. Jack is planting a triangular rose garden. The lengths of two sides of the plot are 8 feet and 12 feet, and the angle between them is 87°. Write an expression that could be used to find the area of this garden?

Law of Sines

Law of sines suggests that in ANY triangle (not just right triangles) the lengths of the sides are proportional to the sines of the corresponding opposite angles.

Formula:

$$\frac{Big\ Letters\ (Angles)}{Little\ Letters\ (sides)} = \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

All are equal! It doesn't matter which ones you use!

Use the law of sines if you are given a triangle with:

Side-Angle-Angle (SAA)
Side-Side-Angle (SSA)- Ambiguous case (MONDAY)

Examples

1. SAA (Side-angle-angle)

2. SAA

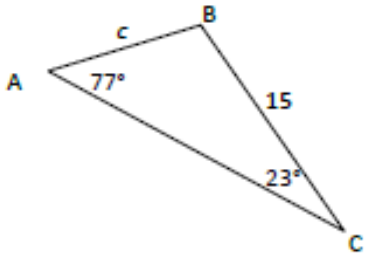
3. SAA

4. A satellite orbiting the earth passes directly overhead at observation stations in Phoenix and Los Angeles, 340 miles apart. At an instant when the satellite is between these two stations, its angle of elevation is simultaneously observed to be 60^0 at Phoenix and 75^0 at Los Angeles. How far is the satellite from Los Angeles

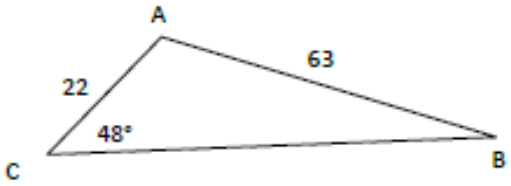
Day 3: Law of Sines Practice

II. Find the length of a side or measure of an angle using Law of Sines.

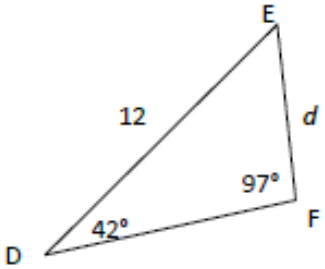
1. For ΔABC find c to the nearest hundredth.



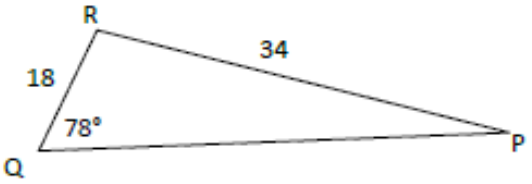
2. For ΔABC find $m\angle B$ to the nearest whole degree.



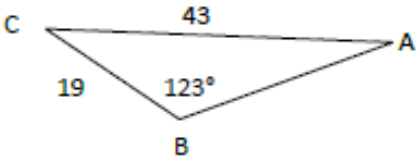
3. For ΔDEF find d to the nearest hundredth.



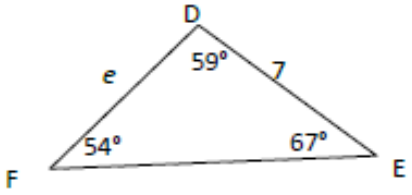
4. For ΔPQR find $m\angle P$ to the nearest whole degree.



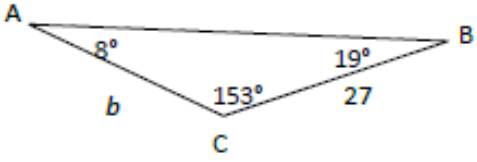
5. For ΔABC find $m\angle A$ to the nearest whole degree.



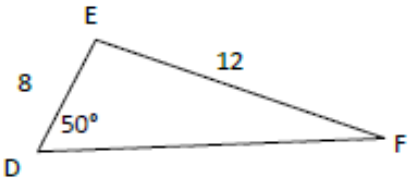
6. For ΔDEF find e to the nearest hundredth.



7. For ΔABC find b to the nearest hundredth.



8. For ΔDEF find $m\angle F$ to the nearest whole degree.



9. For ΔABC, $a = 18$, $b = 6$, and $m\angle A = 28°$. Find $m\angle B$ to the nearest whole degree.

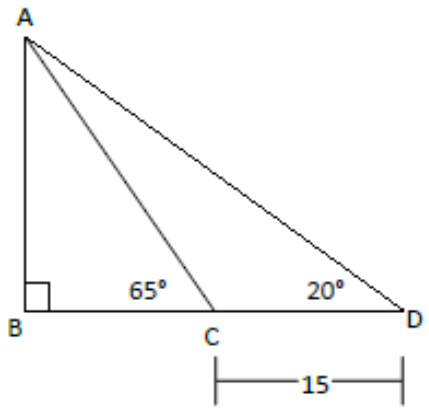
10. For ΔDEF, $d = 24$, $m\angle D = 37°$, and $m\angle E = 49°$. Find e to the nearest whole degree.

11. For ΔDEF, $d = 54$, $f = 27$, $m\angle D = 20°$. Find $m\angle F$ to the nearest whole degree.

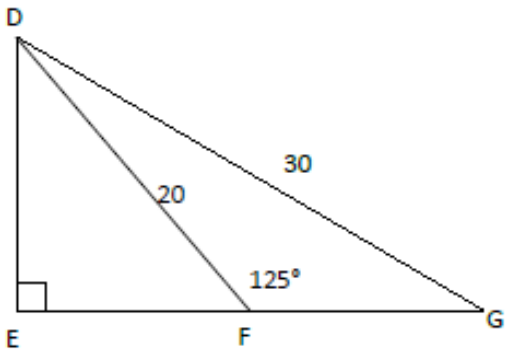
12. For ΔABC, $a = 42$, $c = 72$, and $m\angle C = 41°$. Find $m\angle A$ to the nearest whole degree.

III. Challenge Problems

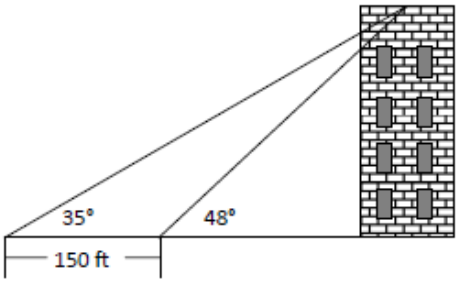
19. For the figure below find BC to the nearest whole number. CD=15.



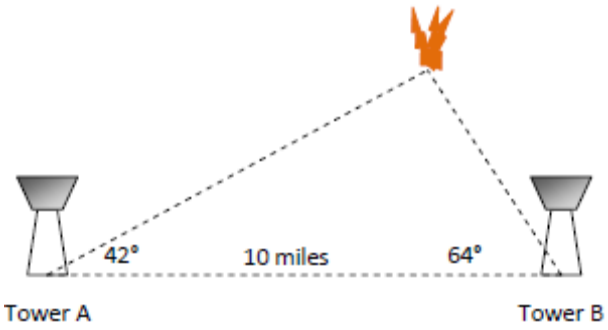
20. For the figure below find $m\angle EDG$ to the nearest whole degree.



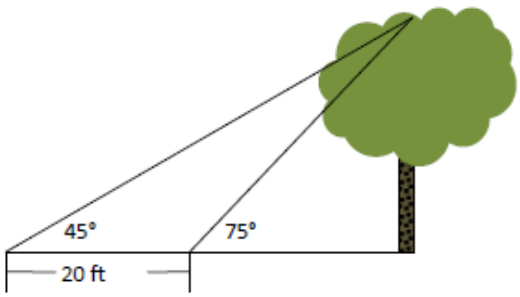
21. Find the height of the building in the figure below to the nearest foot.



22. Fire towers A and B are located 10 miles apart. They use the direction of the other tower as 0°. Rangers at fire tower A spots a fire at 42°, and rangers at fire tower B spot the same fire at 64°. How far from tower A is the fire to the nearest tenth of a mile?

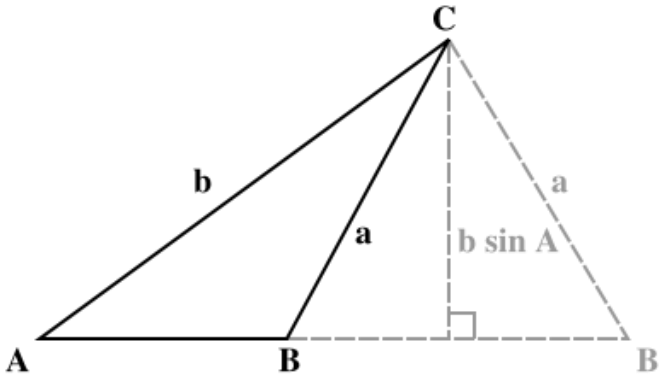


23. Find the height of the tree below to the nearest foot.



The Law of Sines- The Ambiguous Case

If you are given two sides and one angle (SSA), the law of sines could provide 0, 1, or 2 solutions.



To determine how many solutions, add the supplementary angle of the angle found using the law of sines to the given angle

- If when using the law of sines you get an equation where $\sin\ominus > 1$, then there is no solution
- If when using the law of sines you get an equation where $\sin\ominus < 1$, find both the obtuse and the acute angle, then check to see if an obtuse triangles is possible
    - If an obtuse angle is possible, 2 solutions
    - If an obtuse angle is not possible, 1 solution

No Solution Case:

$\angle A = 30, a = 7, c = 16$

1 Solution Case:

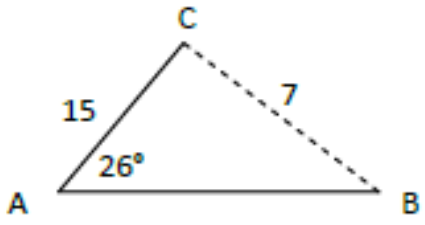
$\angle A = 30, a = 20, c = 16$

2 Solution Case:

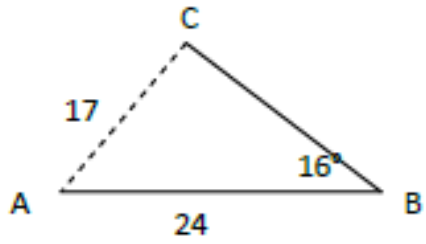
$\angle A = 30, a = 10, c = 16$

II. Practice Problems

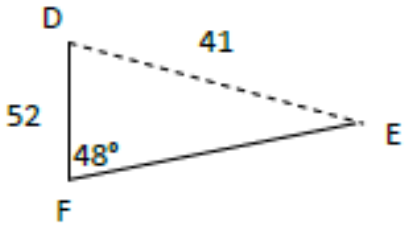
1. For ΔABC, $a = 7, b = 15$, and $m\angle A = 26°$. Find all possible $m\angle B$ to the nearest degree.



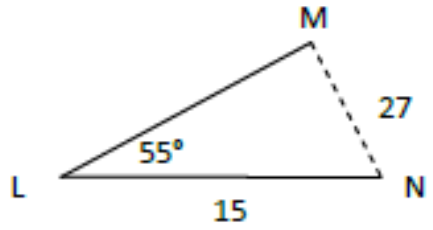
2. For ΔABC, $b = 17, c = 24$, and $m\angle B = 16°$. Find all possible $m\angle C$ to the nearest degree.



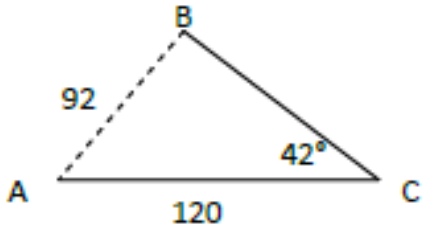
3. For ΔDEF, $e = 52, f = 41$, and $m\angle F = 48°$. Find all possible $m\angle E$ to the nearest degree.



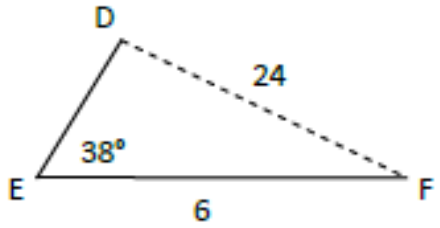
4. For ΔLMN, $l = 27, m = 15$, and $m\angle L = 55°$. Find all possible $m\angle M$ to the nearest degree.



5. For ΔABC, $b = 120, c = 92$, and $m\angle C = 42°$. How many triangles can be formed?



6. For ΔDEF, $d = 6, e = 24$, and $m\angle E = 38°$. How many Triangles can be formed?

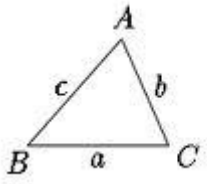


Law of Cosines

Law of Cosines is used when you are given the following information:

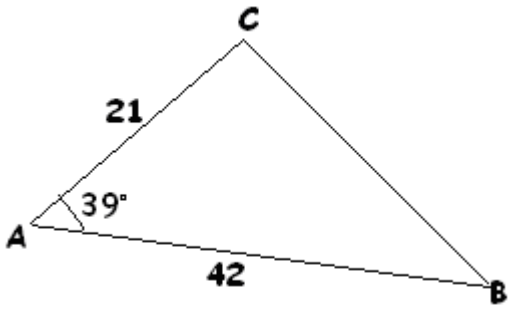
SSS- Know all three sides and no angles

SAS- Know 2 sides and the angle between them

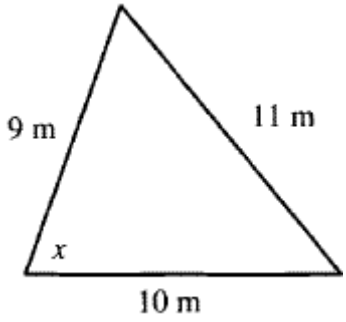


$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = c^2 + a^2 - 2ca\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

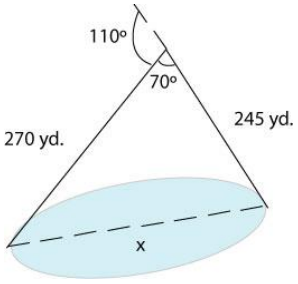
Example 1: In ΔABC, m∠A=39, AC=21 and AB=42. Find side a to the nearest integer.



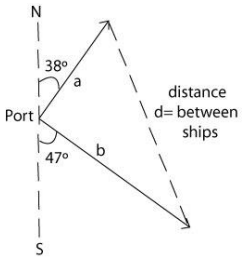
Example 2: In the triangle below, find the measure of angle x.



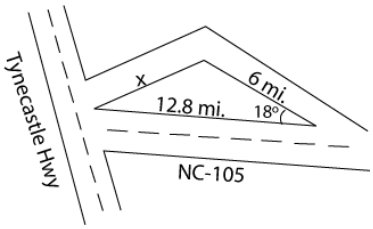
To approximate the length of a lake, a surveyor starts at one end of the lake and walks 245 yards. He then turns 110º and walks 270 yards until he arrives at the other end of the lake. Approximately how long is the lake?



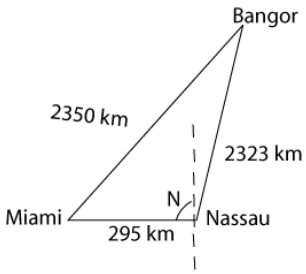
Two ships leave port at 4 p.m. One is headed at a bearing of N 38 E and is traveling at 11.5 miles per hour. The other is traveling 13 miles per hour at a bearing of S 47 E. How far apart are they when dinner is served at 6 p.m.?



You are heading to Beech Mountain for a ski trip. Unfortunately, state road 105 in North Carolina is blocked off due to a chemical spill. You have to get to Tynecastle Highway which leads to the resort at which you are staying. NC-105 would get you to Tynecastle Hwy in 12.8 miles. The detour begins with a 18 veer off onto a road that runs through the local city. After 6 miles, there is another turn that leads to Tynecastle Hwy. Assuming that both roads on the detour are straight, how many extra miles are you traveling to reach your destination?



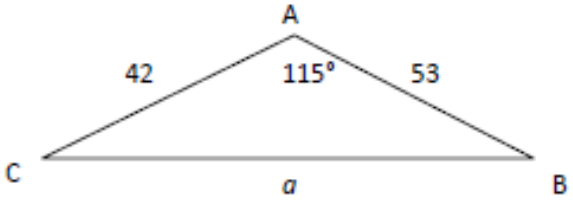
The distance on a map from the airport in Miami, FL to the one in Nassau, Bahamas is 295 kilometers due east. Bangor, Maine is northeast of both cities; its airport is 2350 kilometers from Miami and 2323 kilometers from Nassau. What bearing would a plane need to take to fly from Nassau to Bangor?



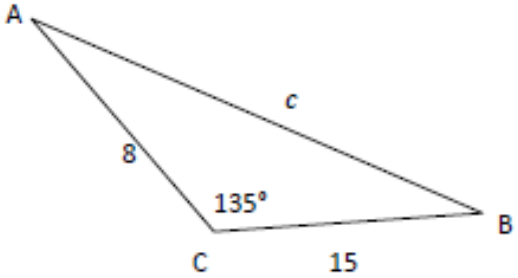
After the hurricane, the small tree in my neighbor's yard was leaning. To keep it from falling, we nailed a 6-foot strap into the ground 4 feet from the base of the tree. We attached the strap to the tree 3½ feet above the ground. How far from vertical was the tree leaning?

II. Find the length of a side using Law of Cosines.

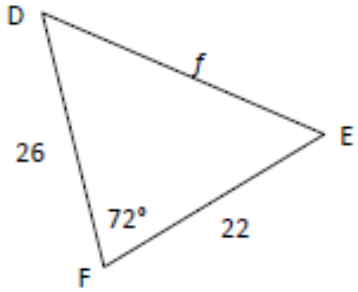
1. For ΔABC find a to the nearest hundredth.



2. For ΔABC find c to the nearest hundredth.



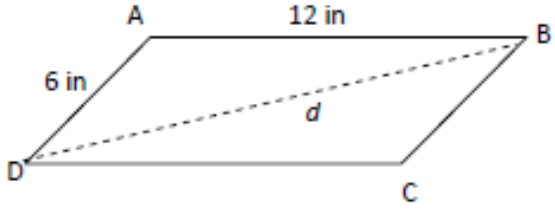
3. For ΔDEF find f to the nearest hundredth.



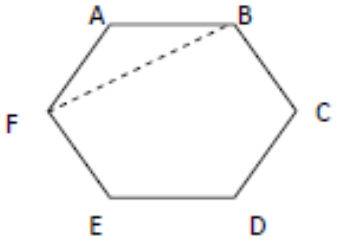
4. For ΔABC find the length of a to the nearest hundredth, given $b = 8, c = 23$, and $m\angle A = 29°$.

5. For ΔABC find the length of c to the nearest hundredth, given $a = 54, b = 47$, and $m\angle C = 85°$.

6. Find the length of the diagonal, d, of the parallelogram below to the nearest inch.

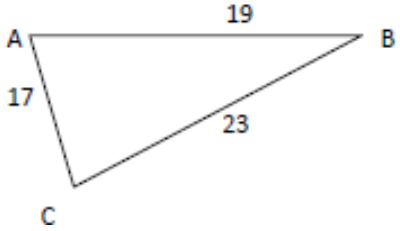


7. A regular hexagon has side lengths of 15 centimeters and angles that measure 120°. Find FB to the nearest centimeter.

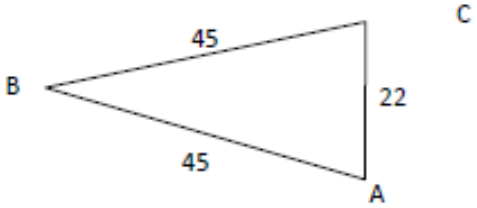


III. Find the measure of an angle using Law of Cosines.

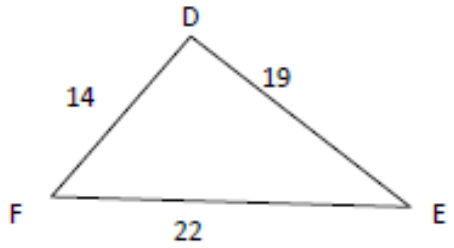
8. For ΔABC find $m\angle A$ to the nearest tenth of a degree.



9. For ΔABC find $m\angle B$ to the nearest tenth of a degree.



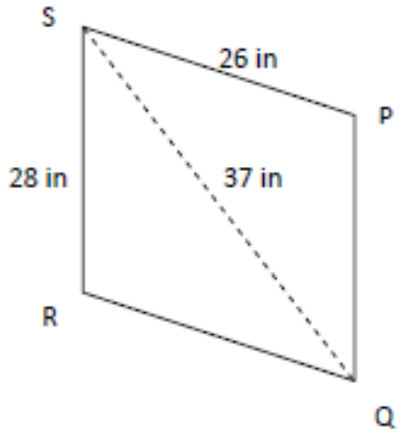
10. For ΔDEF find m∠E to the nearest tenth of a degree.



11. For ΔABC find m∠B to the nearest tenth, given $a = 7, b = 6,$ and $c = 5$.

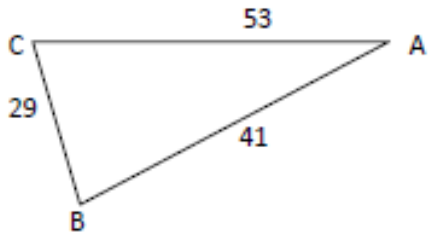
12. For ΔDEF find m∠F to the nearest tenth, given $d = 38, e = 42,$ and $f = 47$.

13. Find m∠P for the parallelogram below to the tenth of a degree.

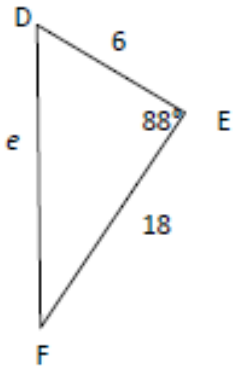


IV. Using Law of Cosines.

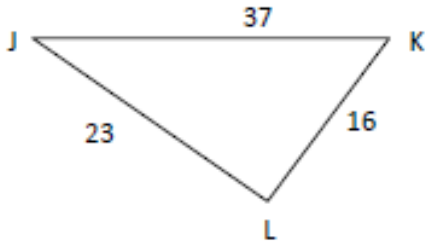
15. For ΔABC find m∠B to the nearest tenth of a degree.



16. For ΔDEF find e to the nearest hundredth.



17. For ΔJKL find m∠K to the nearest tenth of a degree.



18. For ΔXYZ find the length of z to the nearest hundredth, given $x = 81, y = 75,$ and $m\angle Z = 42°$.

Law of Sines/Cosines Word Problems

1. A post is supported by two wires (one on each side going in opposite directions) creating an angle of 80° between the wires. The ends of the wires are 12m apart on the ground with one wire forming an angle of 40° with the ground. Find the lengths of the wires.

2. Two ships are sailing from Halifax. The Nina is sailing due east and the Pinta is sailing 43° south of east. After an hour, the Nina has travelled 115km and the Pinta has travelled 98km. How far apart are the two ships?

3. 3 friends are camping in the woods, Bert, Ernie and Elmo. They each have their own tent and the tents are set up in a Triangle. Bert and Ernie are 10m apart. The angle formed at Bert is 30°. The angle formed at Elmo is 105°. How far apart are Ernie and Elmo?

4. Two scuba divers are 20m apart below the surface of the water. They both spot a shark that is below them. The angle of depression from diver 1 to the shark is 47° and the angle of depression from diver 2 to the shark is 40°. How far are each of the divers from the shark?

5. To estimate the length of a lake, Caleb starts at one end of the lake and walks 95m. He then turns and walks on a new path, which is 120° to the direction he was first walking in, and walks 87m more until he arrives at the other end of the lake. Approximately how long is the lake?

6. Two observers are standing on shore ½ mile apart at points F and G and measure the angle to a sailboat at a point H at the same time. Angle F is 63° and angle G is 56°. Find the distance from each observer to the sailboat.

7. Jack and Jill both start at point A. They each walk in a straight line at an angle of 105° to each other. After 45 minutes Jack has walked 4.5km and Jill has walked 6km. How far apart are they?

8. Points A and B are on opposite sides of the Grand Canyon. Point C is 200 yards from A. Angle B measures 87° and angle C measures 67°. What is the distance between A and B?

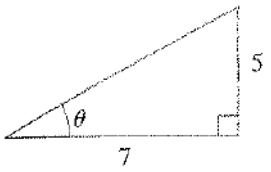
9. A 4m flag pole is not standing up straight. There is a wire attached to the top of the pole and anchored in the ground. The wire is 4.17m long. The wire makes a 68° angle with the ground. What angle does the flag pole make with the wire?

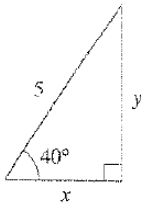
Unit 2 Review

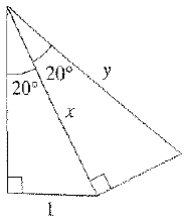
7.2 Things to Know!

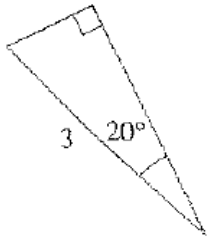
- Solve a right triangle, SOH CAH TOA,
  $\sin\theta = \frac{opp}{hyp}, \cos\theta = \frac{adj}{hyp}, \tan\theta = \frac{opp}{adj}, \csc\theta = \frac{hyp}{opp}, \sec\theta = \frac{hyp}{adj}, \cot\theta = \frac{adj}{opp}$
- Use SOH CAH TOA to solve RIGHT triangles. (Problems that say angle of elevation/depression)

Practice:

1. 

2. 

3. 

4. 

5. A guy wire from the top of the transmission tower at WJBC forms a 75° angle with the ground at a 55-foot distance from the base of the tower. How tall is the tower?

6. The base of a ladder is 6ft from the building, and the angle formed by the ladder and the ground is 73°. How high up the building does the ladder touch?

7.4 Things to Know!

- Law of Sines - $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.
- Use if you have ASA or AAS (that is not a right triangle), then you will only produce 1 triangle
- Use if you have SSA (that is not a right triangle), then you could produce 0, 1 or 2 triangles. If sin A>1, then no solution. If sin A < 1, consider 2 triangles!!!!

Practice:

7. Two markers A and B are on the same side of a canyon rim 56 ft apart. A third marker, C, located across the rim, is positioned so that ΔBAC = 72° and ΔABC = 53°. Find the distance between C and A.

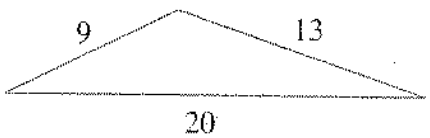
8. A civil engineer wants to determine the distances from points A and B to an inaccessible point C, as shown. From direct measurements, the engineer knows that AB = 25m, ∠A = 110°, and ∠B = 20°. Find AC and BC.

7.5 Things to Know!

- Law of Cosines - $a^2 = b^2 + c^2 - 2bc\cos A$, $b^2 = a^2 + c^2 - 2ac\cos B$, $c^2 = a^2 + b^2 - 2ab\cos C$
- Use Law of Cosines if you have SAS or SSS.

Practice:

9. Find the measure of the largest angle in the triangle below.



10. In order to determine the distance between two points A and B on opposite sides of a lake, a surveyor chooses a point C that is 900 ft from A and 225 ft from B. If the measure of the angle at C is 70°, find the distance between A and B.

11. A car travels along a straight road, heading east for 1 hour, then changing to northeast direction at 135º onto another road, traveling for 30 min. If the car has maintained a constant speed of 40mph, how far is it from its starting point?

12. Suppose you want to fence a triangular lot. If two sides measure 84 feet and 78 feet and the angle between the two sides is 102º, what is the length of the fence to the nearest foot?

Area of a Triangle

- The area of a triangle with sides of lengths a and b and with included angle θ is $A = \frac{1}{2}ab\sin\theta$.

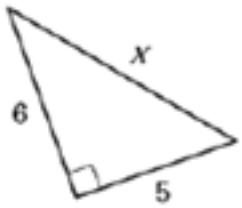
Practice:

13. Find the area of a triangle whose side lengths are 8 and 14 and has an included angle of 35º.

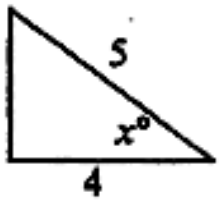
14. Find the area of a triangle with side lengths 5, 6 and 8.

Mixing it all up…

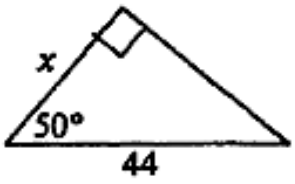
15. Solve for x



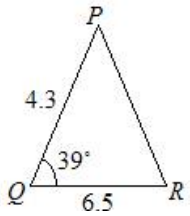
16. Solve for x



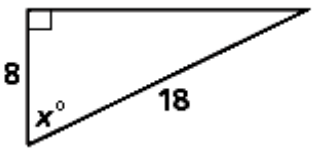
17. Solve for x



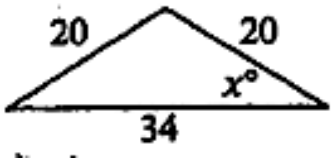
18. Find the area of the ΔPQR



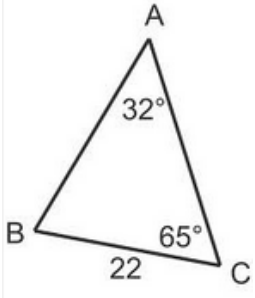
19. Solve for x



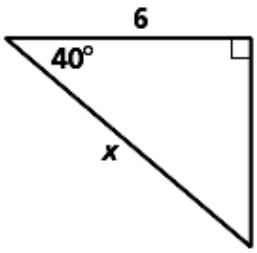
20. Solve for x



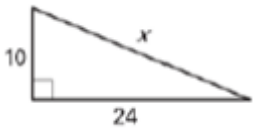
21. Find the length of side AB



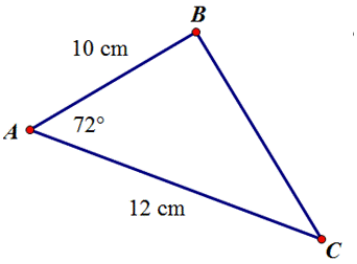
22. Solve for x



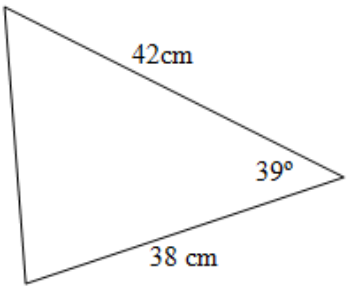
23. Solve for x



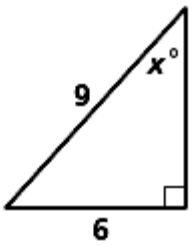
24. Find the area of ΔABC.



25. Solve for the missing side



26. Solve for x



27. From the top of a 120 foot tower, an air traffic controller observes an airplane on the runway at an angle of depression of 19°. How far from the base of the tower is the airplane?

28. Find the angle of elevation of the sun when a 12.5 meter tall telephone pole casts an 18 meter long shadow.

29. If $\tan\theta=8/17$, find the other 6 trig ratios

30. If $\csc\theta=\sqrt{13}/4$, find the other 6 trig ratios

31. If $\cos(x)=0.42$, what is the measure of angle x?

32. Evaluate tan(45)

33. Find the area of triangle ABC if angle A is 30 degrees, AB=12 and AC=14.

34. In triangle ABC, if a=6, b=10 and ∠A=42, how many triangles can be formed?

35. From a point A on the ground, the angle of elevation to the top of a tall building is 24.1°. From a point B, which is 600 feet closer to the building, the angle of elevation is measured to be 30.2°. Find the height of the building.