# Unit 4: Exponential and Logarithmic Functions

1.01a
Create and use calculator-generated models for linear, polynomial, exponential, trigonometric, power, and logarithmic functions of bivariate data to solve problems. Interpret the constants, coefficients, and bases in the context of the data.
1.01b
Create and use calculator-generated models for linear, polynomial, exponential, trigonometric, power, and logarithmic functions of bivariate data to solve problems. Check models for goodness-of-fit; use the most appropriate model to draw conclusions and make predictions.
2.01a
Use logarithmic (common, natural) functions to model and solve problems; justify results. Solve using tables, graphs, and algebraic properties.
2.01b
Use logarithmic (common, natural) functions to model and solve problems; justify results. Interpret the constants, coefficients, and bases in the context of the problem.

| Day | Topic | Activity |
|---|---|---|
| 1 | Introduce exponential functions (review) | Textbook Pg. 384-385 #2-5, #11-29 odd, #39, #53<br>Pg, 405 #1-4, 7-9 |
| 2 | -Discuss ‘e’ and where it comes from<br>-Word problems containing ‘e’ (compounding interest, pop growth, not finding exponents yet)<br>-Graphs of ‘e’ | -Class Notes on word problems |
| 3 | -Exp to Log and Log to Exp (switching back and forth)<br>-Properties of Logarithmic Functions | |
| 4 | -Solving for exponents, taking logs of both sides | |
| 5 | Log rules | **Quiz** |
| 6 | -Continue with word problems, using logs to solve<br>-Work on CSI Project | |
| 7 | **PSAT** | **PSAT** |
| 8 | Review | |
| 9 | CSI PROJECT DAY | |
| 10 | **TEST** | **TEST** |

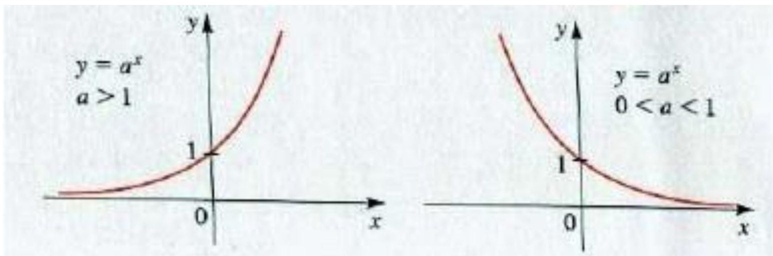
CSI Project Due ______________________

## Day 1 Notes: Exponential Functions

**Exponential Function** is a function in the form $a^x$, where $a > 0, a \neq 1$. $a$ is referred to as the base.

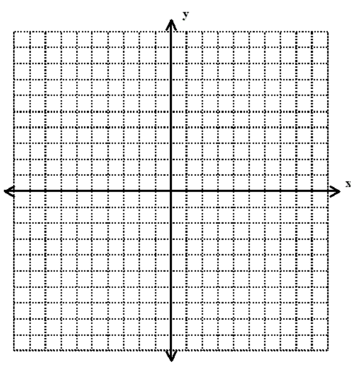
Important Notes:

1.
2.
3.
4.
5.

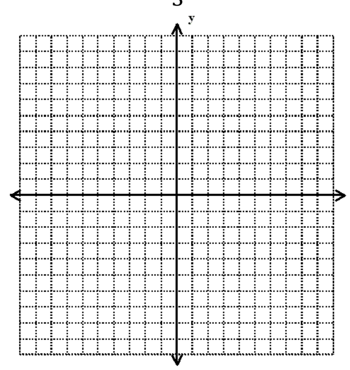



Graph the following functions and identify if they are exponential growth or decay.

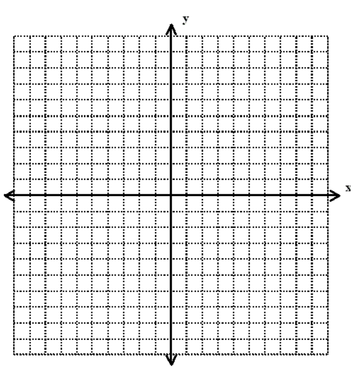
a) $f(x) = 3^x$

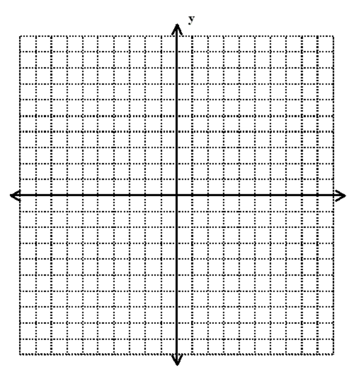


b) $f(x) = \frac{1}{3}^x$



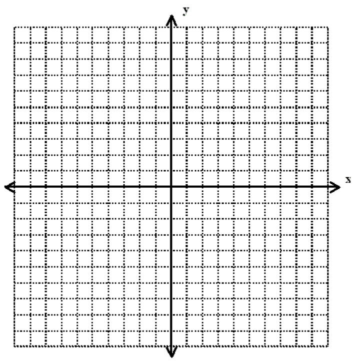
Find the exponential function whose graph is given

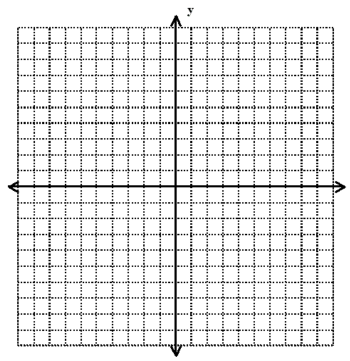


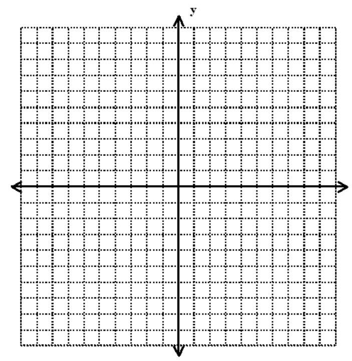


The same transformation rules follow with the exponential functions as well.
Graph the following functions using the transformation rules.

a) $g(x) = 1 + 2^x$
b) $h(x) = -2^x$
c) $c(x) = 10^{x-1}$




**Horizontal Asymptotes**

a) $f(x) = 2^x + 1$
b) $g(x) = 3^x - 5$
c) $k(x) = 4 + 2^{x-1}$
d) $p(k) = -2^k + 3$

**Logarithmic Functions**

Every exponential function has an inverse function called the ____________________________

Let a be a positive number with $a \neq 1$. The logarithmic function with base a, denoted $\log_a$ is defined by

$$log_a x = \qquad\qquad =$$

In words, this says, $\log_a x$ is the exponent to which the base a must be raised to give x.

**Switching between logarithmic form and exponential form**

**Complete the following chart:**

| Log Form | Exp. Form |
|---|---|
| $log_{10}100{,}000 = 5$ | |
| | $2^3 = 8$ |
| $log_2\left(\frac{1}{8}\right) = -3$ | |
| | $5^r = s$ |

**Graphing Log Functions**

Since the log function is just the inverse of the exponential function, it reflects the graph over the line y=x).

Graph: $f(x) = log_2 x$

1. If we were to rewrite this in exponential form we would get $x = 2^y$
   To make a table of values, we should only pick x values that are powers of 2.

| x | y |
|---|---|
| | |
| | |
| | |
| | |
| | |
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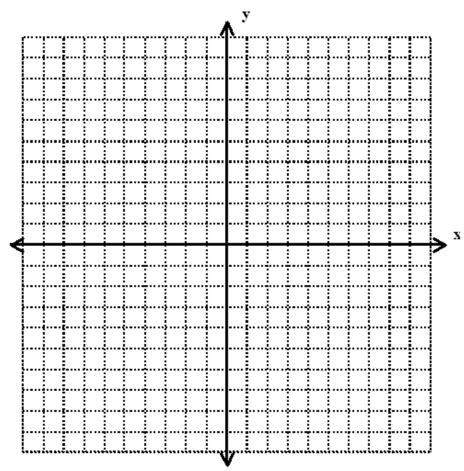
| | |

2. Now we can plot these points and graph the function.




Note: The logarithm with base 10 is called the **common log** and is denoted by omitting the base $\log x = log_{10}x$

## Day 1 Practice

1. Write the following in exponential form:

(a) $\log_3 x = 9$

(b) $\log_2 8 = x$

(c) $\log_3 27 = x$

(d) $\log_4 x = 3$

(e) $\log_2 y = 5$

(f) $\log_5 y = 2$

2. Write the following in logarithm form:

(a) $y = 3^4$

(b) $27 = 3^x$

(c) $m = 4^2$

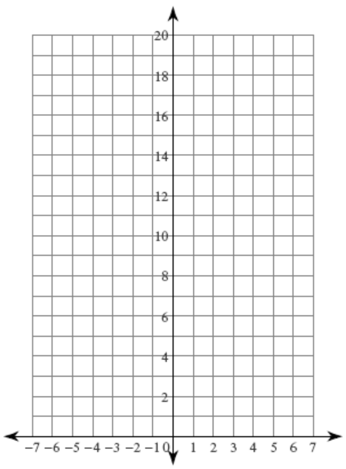
(d) $y = 3^5$

(e) $32 = x^5$

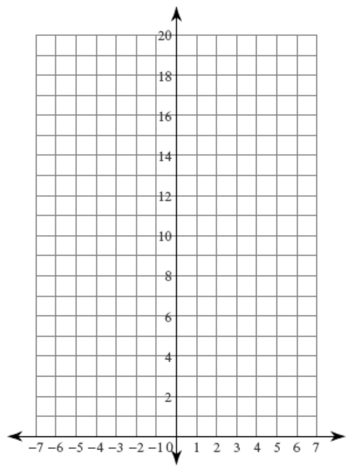
(f) $64 = 4^x$

**Sketch the graph of each function.**

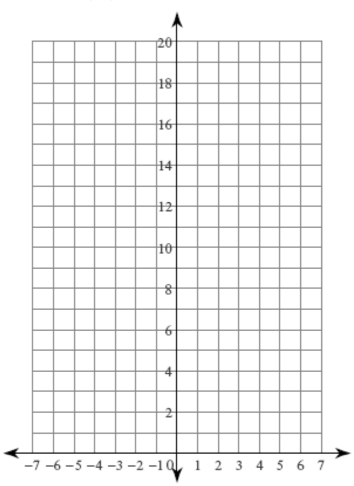
1) $y = 2 \cdot 3^x$



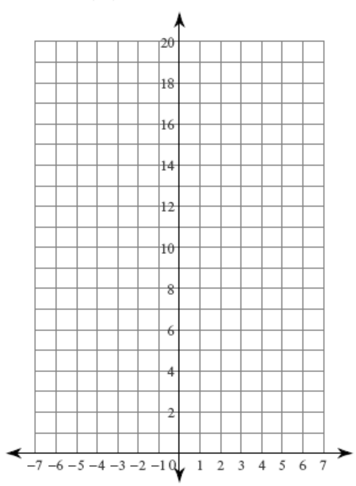

2) $y = 5 \cdot 2^x$




3) $y = 4 \cdot \left(\frac{1}{2}\right)^x$

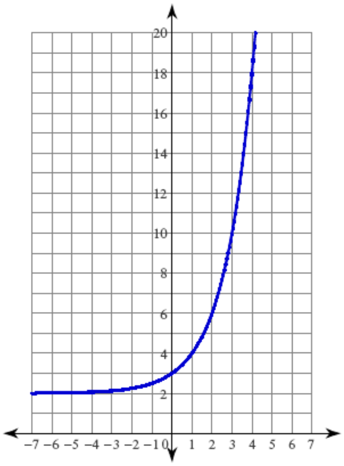



4) $y = 2 \cdot \left(\frac{1}{2}\right)^x$



**Write an equation for each graph.**

7)

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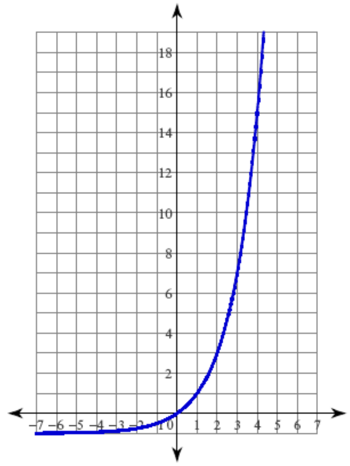
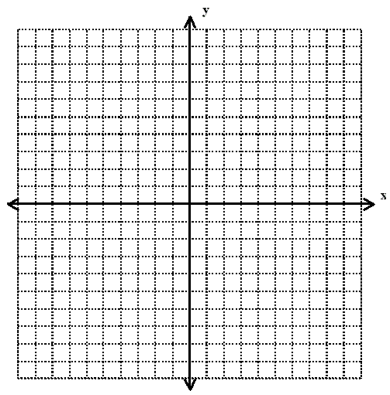
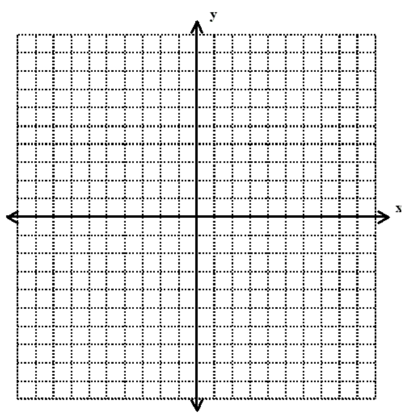



**Graph the following logarithmic functions. Try making a table if you are stuck.**

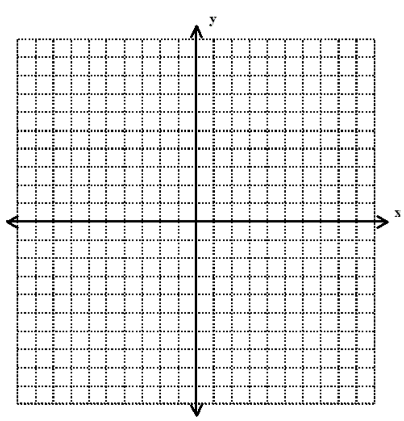
1. $f(x) = log_3 x$




2. $f(x) = log_{10} x$




3. $f(x) = log_4 x$



## Day 2 Notes: The Natural Exponential Function

"Euler's number," or e, is a constant (similar to pi). "e" is approximately 2.71828. It is the limit of $(1 + 1/n)^n$ as $n$ becomes large.

The **natural exponential function** is the exponential function $f(x) = e^x$ with base $e$.

We can use e for several types of word problems.

**Compound Interest**

Compound interest is interest compounded after different amounts of time periods, therefore becoming an exponential function.

Compound Interest is calculated by the formula

A(t)=

P=

r =

n =

t =

Key words:

Yearly= Monthly = Quarterly = Semi-annually= Daily=

Ex 1: A sum of $1000 is invested at an interest rate of 12% per year. Find the amounts in the account after 3 years if interest is compounded annually, semiannually, quarterly, monthly and daily

Ex 2: How much would you have to invest in order to have $700 after 2 years if the interest is compounded quarterly at an interest rate of 7.5%?

**Continuously Compounded Interest**

$A(t) =$
$P=$
$r =$
$t =$

Ex 1: Find the amount after 3 years if $1000 is invested at an interest rate of 12% per year, compounded continuously.

Ex 2: Find the initial amount invested if after 3 years at an interest rate of 5.75% compounded continuously, you have $7500.

**Exponential Growth/Decay**

A population that experiences exponential growth increases according to the model

$n(t)=$
$n_0=$
$r=$
$t=$

Ex 1: The initial bacterium count in a culture is 400. A biologist later makes a sample count of bacteria in the culture and finds that the relative rate of growth is 40% per 2 hours.

a) Find a function that models the number of bacteria after t hours
b) What is the estimated count after 8 hours?

Ex 2: A certain breed of rabbit was introduced into a small island about 8 years ago. The current rabbit population on the island is estimated to be 4100, with a relative growth rate of 55% per year. What was the initial size of the rabbit population?

## Half Life

If $m_0$ is the initial mass of a radioactive substance with half-life h, then the mass remaining at time t is modeled by the function

Where $r =$

***UNITS OF TIME ARE THE SAME THROUGHOUT THE PROBLEM!

Ex 1: Polonium-210 has a half life of 140 days. Suppose a sample of this substance has a mass of 300 mg.

a) Find a function that models the amount of the sample remaining at time t

b) Find the mass remaining after one year

Ex 2: The half-life of a radioactive substance is one hundred fifty-three days. How many days will it take for seventy percent of the substance to decay?

## Practice:

1. William wants to have a total of $4000 in two years so that he can put a hot tub on his deck. He finds an account that pays 5% interest compounded monthly. How much should William put into this account so that he'll have $4000 at the end of two years?

2. Kelly plans to put her graduation money into an account and leave it there for 4 years while she goes to college. She receives $750 in graduation money that she puts it into an account that earns 4.25% interest compounded semi-annually. How much will be in Kelly's account at the end of four years?

3. If interest is compounded continuously at 4.5% for 7 years, how much will a $2000 investment be worth at the end of 7 years?

4. If \$8000 is invested in an account that pays 4% interest compounded continuously, how much is in the account at the end of 10 years?

5. If a \$500 certificate of deposit earns 4.25% compounded continuously then how much will be accumulated at the end of a 3 year period?

6. A certain investment earns 8.75% compounded continuously. If \$10,000 dollars is invested for 5 years how much will be in the account after 5 years?

7. A certain bacterium has an exponential growth rate of 25% per day. If we start with 0.5 gram and provide unlimited resources how much bacteria can we grow in 2 weeks?

8. The half-life of chromium-51 is 28 days. If the sample contained 510 grams, how much chromium would remain after 56 days? How much would remain after 1 year?

9. Titanium-51 decays with a half life of 6 minutes. What fraction of titanium would remain after one hour?

# Logarithms

The logarithm with base a of a positive number x is defined as:

For $x > 0$ and $0 < a \neq 1$,
$y = \log_a x$ if and only if $x = a^y$

In functional notation:
$f(x) = \log_a x$ is called the logarithmic function with base a

Because the logarithm with base e is used so frequently it has been given a special name **Natural logarithm** and abbreviation ln. The function is defined as:

$f(x) = \log_e x = \ln x \quad x > 0$ is the natural logarithmic function.

By definition, the natural log function has an inverse function which is an exponential function.

$$\ln x = y \leftrightarrow e^y = x$$

## Properties of Logarithms

| | | |
|---|---|---|
| 1. | $\log_a 1 = 0$ | because $a^0 = 1$ |
| 2. | $\log_a a = 1$ | because $a^1 = a$ |
| 3. | $\log_a a^x = x$ | Inverse Properties |
| 4. | $a^{\log x} = x$ | Inverse Properties |

## Properties of Natural Logarithms

All of the properties of logarithms listed above work for Natural Logarithms. There are also some special properties that apply only to the Natural Logarithms.

| | | |
|---|---|---|
| 1. | $\ln 1 = 0$ | because $e^0 = 1$ |
| 2. | $\ln e = 1$ | because $e^1 = e$ |
| 3. | $\ln e^x = x$ | Inverse Properties |
| 4. | $e^{\ln x} = x$ | Inverse Properties |

**Examples:**
Evaluate the following:

a) $log_5 1 =$
b) $log_5 5^8 =$
c) $log_5 5 =$
d) $5^{log_5 12} =$
e) $\ln e^8 =$
f) $\ln\left(\frac{1}{e^2}\right) =$
g) $\ln 5 =$

**Practice:** Use the definition of logs to find x

a. $\log_3 x = 4$
b. $\log_x 125 = 3$
c. $\log_6 36 = x$
d. $\log_4 x = \frac{1}{2}$
e. $\log_3\left(\frac{1}{9}\right) = x$
f. $\log_3 243 = x$

Evaluate

a. $\log 1$
b. $\log_4 4$
c. $\log_6 6^5$
d. $\log_4 2$
e. $10^{\log 7}$
f. $\log_9\left(\frac{1}{3}\right)$
g. $\ln e^3$

**Laws of Logarithms**

| Property | Definition | Example |
|---|---|---|
| Product | $\log_b mn = \log_b m + \log_b n$ | $\log_3 9x = \log_3 9 + \log_3 x$ |
| Quotient | $\log_b \frac{m}{n} = \log_b m - \log_b n$ | $\log_{\frac{1}{4}} \frac{4}{5} = \log_{\frac{1}{4}} 4 - \log_{\frac{1}{4}} 5$ |
| Power | $\log_b m^p = p \cdot \log_b m$ | $\log_2 8^x = x \cdot \log_2 8$ |
| Equality | If $\log_b m = \log_b n$,, then *m = n*. | $\log_8(3x-4) = \log_8(5x+2)$ so, 3x – 4 = 5x+2 |

Examples: Use the laws of logs to rewrite each expression

a) $log_2(6x)$
b) $log\sqrt{5}$
c) $log_5(x^3y^6)$
d) $\ln(\frac{ab}{\sqrt[3]{c}}$

Use the laws of logs to evaluate each expression

a) $log_4(2) + log_4(32)$
b) $log_2(80) + log_2(5)$
c) $-\frac{1}{3}log\quad 8$

# Properties of Logs

(09 br)

Write each equation in ***exponential*** form.

1. $\log_2 64 = 6$

2. $\log_4 \frac{1}{64} = -3$

3. $\log_{10}(0.01) = -2$

Write each equation in ***logarithmic*** form.

4. $2^5 = 32$

5. $5^{-1/2} = \frac{\sqrt{5}}{5}$

6. $10^{-1} = 0.1$

***Evaluate*** the expression. Hint—set = x and solve for x.

7. $\log_2 8$

8. $\log_8 64$

9. $\log_6 216$

10. $\log_7 7$

11. $\log_6 1$

12. $\log_8 \frac{1}{8}$

13. $\log_7 \frac{1}{49}$

14. $\log_9 \frac{1}{27}$

15. $\log_5 \sqrt{5}$

16. $\log_9 3$

17. $\log_2 16$

18. $\log_{1/2} 16$

Solve for x.

19. $\log_6 x = 2$

20. $\log_5 x = 3$

21. $\log_{16} x = -1$

22. $\log_9 x = 2$

23. $\log_{1/4} x = -2$

24. $\log_x 64 = 3$

25. $\log_x 8 = -1$

## Log Rules Practice

**Expand each logarithm.**

1) $\log(6 \cdot 11)$

2) $\log(5 \cdot 3)$

3) $\log\left(\frac{6}{11}\right)^5$

4) $\log(3 \cdot 2^3)$

5) $\log\frac{2^4}{5}$

6) $\log\left(\frac{6}{5}\right)^6$

7) $\log\frac{x}{y^6}$

8) $\log(a \cdot b)^2$

9) $\log\frac{u^4}{v}$

10) $\log\frac{x}{y^5}$

11) $\log\sqrt[3]{x \cdot y \cdot z}$

12) $\log(x \cdot y \cdot z^2)$

**Condense each expression to a single logarithm.**

13) $\log 3 - \log 8$

14) $\dfrac{\log 6}{3}$

15) $4\log 3 - 4\log 8$

16) $\log 2 + \log 11 + \log 7$

17) $\log 7 - 2\log 12$

18) $\dfrac{2\log 7}{3}$

19) $6\log_3 u + 6\log_3 v$

20) $\ln x - 4\ln y$

21) $\log_4 u - 6\log_4 v$

22) $\log_3 u - 5\log_3 v$

23) $20\log_6 u + 5\log_6 v$

24) $4\log_3 u - 20\log_3 v$

**Critical thinking questions:**

25) $2(\log 2x - \log y) - (\log 3 + 2\log 5)$

26) $\log x \cdot \log 2$

## Solving Logarithmic Equations

Guidelines for solving exponential equations

Example:

a) Find the solution of the equation $-2=\log(2)-\log(3+x)$

b) Find the solution to the equation $\log_4(x-4)+\log_4 x=\log_4 5$

c) Find the solution to the equation $\ln 2x+\ln 4=3$

**Practice:**

1) $\log_2(x+2)+\log_2 5=4$

2) $\log_4(2x+1)-\log_4(x-2)=1$

3) $\log_6(x+2)=2$

4) $\log_3 x+\log_3(x-2)=1$

**Solve each equation.**

1) $\log 5x = \log (2x + 9)$

2) $\log (10 - 4x) = \log (10 - 3x)$

3) $\log (4p - 2) = \log (-5p + 5)$

4) $\log (4k - 5) = \log (2k - 1)$

5) $\log (-2a + 9) = \log (7 - 4a)$

6) $2\log_7 -2r = 0$

7) $-10 + \log_3 (n + 3) = -10$

8) $-2\log_5 7x = 2$

9) $\log -m + 2 = 4$

10) $-6\log_3 (x - 3) = -24$

11) $\log_{12} (v^2 + 35) = \log_{12} (-12v - 1)$

12) $\log_9 (-11x + 2) = \log_9 (x^2 + 30)$

13) $\log(16+2b)=\log(b^2-4b)$

14) $\ln(n^2+12)=\ln(-9n-2)$

15) $\log x+\log 8=2$

16) $\log x-\log 2=1$

17) $\log 2+\log x=1$

18) $\log x+\log 7=\log 37$

19) $\log_8 2+\log_8 4x^2=1$

20) $\log_9(x+6)-\log_9 x=\log_9 2$

21) $\log_6(x+1)-\log_6 x=\log_6 29$

22) $\log_5 6+\log_5 2x^2=\log_5 48$

23) $\ln 2-\ln(3x+2)=1$

24) $\ln(-3x-1)-\ln 7=2$

25) $\ln(x-3)-\ln(x-5)=\ln 5$

26) $\ln(4x+1)-\ln 3=5$

## Solving Exponential Equations

Guidelines for solving exponential equations

1. Isolate the exponential expression on one side of the equation
2. Take the log of each side, then use the Laws of Logs to "bring down the exponent"
3. Solve for the variable

Example:

c) Find the solution of the equation $3^{x+2} = 7$

d) Find the solution to the equation $8e^{2x} = 20$

**Solve each equation. Round your answers to the nearest ten-thousandth.**

1) $3^{b} = 17$

2) $12^{r} = 13$

3) $9^{n} = 49$

4) $16^{v} = 67$

5) $3^{a} = 69$

6) $6^{r} = 51$

7) $6^{n} = 99$

8) $20^{r} = 56$

13) $16^{n-7} + 5 = 24$

14) $20^{-6n} + 6 = 55$

15) $5 \cdot 6^{3m} = 20$

16) $8^{-5a} - 5 = 53$

17) $-2 \cdot e^{-3n-8} - 3 = -45$

18) $10 \cdot e^{2n-10} - 5 = 73$

19) $10 \cdot e^{10-3m} - 8 = 23$

20) $-8 \cdot e^{5-6x} + 10 = -16$

21) $-5.1 \cdot e^{10n+2} + 7.9 = 2$

22) $5.1 \cdot e^{10b+8} - 7 = 76.4$

23) $3 \cdot e^{4-3x} + 1.6 = 45.6$

24) $-2 \cdot e^{9.4x-5} + 6 = -68.9$

**Day 6: Word Problems**

1. How long does it take for an investment to double in value if it is invested at 8% per annum compounded monthly? Compounded continuously?

2. If Akul has $100 to invest at 8% per annum compounded monthly, how long will it be before he has $150? If the compounding is continuous, how long will it be?

3. How many years will it take for an initial investment of $10,000 to grow to $25,000? Assume a rate of interest of 6% compounded continuously

4. Sears charges 1.25% per month on the unpaid balance for customers with charge accounts (compounded monthly). A customer charges $200 and does not pay her bill for 6 months. What is the bill at that time?

5. Rupurt will be buying a new car for $15000 in three years. How much money should he ask his parents for now so that, if he invests it at 5% compounded continuously, he will have enough to buy a new car?

6. A business purchased for $650000 in 1994 is sold in 1997 for $850000. What is the annual rate of return for this investment?

7. Tanya has just inherited a diamond ring appraised at $5000. If diamonds have appreciated in value at an annual rate of 8%, what was the value of the ring 10 years ago when the ring was purchased?

8. On January 1, Kim places $1000 in a certificate of deposit that pays 6.8% compounded continuously and matures in 3 months. Then Kim places the $1000 and the interest in a passbook account that pays 5.25% compounded monthly. How much does Kim have in the passbook account on May 1?

9. The population of a colony of mosquitoes obeys the law of uninhibited growth. If there are 1000 mosquitoes initially, and there are 1800 after one day, what is the size of the colony after 3 days? How long is it till there are 10000 mosquitoes?

10. The population of a southern city follows the exponential law. If the population doubled in size over an 18-month period and the current population is 10000, what will be the population 2 years from now?

11. The half-life of Radium is 1690 years. If ten grams are present now, how much will be present in 50 years?

12. A piece of charcoal is found to contain 30% of the carbon-14 it originally had. When did the tree from which the charcoal came die? Use 5600 years as the half-life of carbon-14.

13. After the release of radioactive material into the atmosphere in Ukraine in 1986, the hay in Austria was contaminated by iodine-131 (half life 8 years). If it is okay to feed the hay to cows when 10% of the iodine-131 remains, how long do the farmers need to wait to use this hay?

14. The size of P of a certain insect population at time t (in days) obeys the equation $P = 500e^{0.02t}$. After how many days will the population reach 1000? When will it reach 2000?

15. The half-life of radium is 1690 years. If 10 grams are present now, how much will be present in 50 years?

## AFM Unit 3 Exp and Log Test – Study Guide

### *Complete #1-25 all. For #26-100, pick and choose questions to do as needed.

**I. Write the equation in exponential form. Do NOT evaluate.**

1. $\log_5 47 = x$ ________________ 2. $\ln 9 = z$ _______________

**II. Write the equation in logarithmic form. Do NOT evaluate.**

3. $7^3 = x$ ___________________ 4. $e^{5x} = k$ ________________

**III. Evaluate each expression. Write your answer in the blank space. Round your answer to 2 decimal places.**

5. $\log_4 \sqrt{36}$ 6. $\log_3 25$ 7. $5^{\log_5 30}$

5. ____________ 6. ____________ 7. ______________

8. ln 21 9. $\log_4 1$ 10. Which is larger, $\log_3 300$ or $\log_6 800$ ?

8. ____________ 9. ____________ 10. ____________

**IV. Solve each equation. Show your work. Round your answer to 2 decimal places.**

11. $5^{4x+6} = 23$ 12. $e^{.26t} = 68$ 13. $20e^{-.4t} = 160$

11. ______________ 12. ______________ 13. ______________

14. $3^{6x-3} = 6$ 15. log (t – 20) = 6 16. 3 ln (2 – x) = 9

14. ______________ 15. ______________ 16. ______________

**VI. Solve the following problems. Be sure to first include the formula you will use for the problem, then show all of your work! Round your answer to 2 decimal places. Please circle or box your final answer!**

17. The number of a certain species of fish is modeled by the function $n(t) = 23e^{0.053t}$ where t is measured in years and n(t) is measured in millions.
    a) Express as a percentage, the relative rate of growth of the fish population. ________
    b) What was the initial fish population? ____________
    c) What will the fish population be after 7 years? Show your work! ______________

18. Ashley will be buying a car for $24000 in five years. How much money should she ask her parents for now so that, if she invests at 8.2% compounded continuously, she will have enough to buy the new car?

19. Suppose that $6,045 is invested in a savings account paying 7.25% interest per year.
    a) Write the formula for the amount in the account after *t* years if interest is compounded semiannually. ________________

    b) Find the amount in the account after 5 years. _____________________

    c) How long will it take for the amount in the account to grow to $9,500?

20. A sum of $1475 was invested for six years and the interest was compounded monthly. If this sum amounted to $4832.46 after the given time, what was the interest rate? Show your work!

21. Find a function that models the rabbit population in a North Carolina county after 1996. Assume that the population grows exponentially. In 1996, the population was 20,000 rabbits and in 2000 the population was 53,000. In what year will the population reach 100,000?

22. The half life of radium-226 is 5 days. After 25 days a sample has been reduced to 0.375 g.

    a) What was the initial mass of the sample?

    b) After how many days will only 0.15 g remain?

23. What interest rate is required for an investment with continuously compounded interest to double in 20 years?

24. How long does it take for an investment to double in value if it is invested at 5% compounded weekly (n=52)?

25. If the interest rates are the same, would you choose a savings account that compounded interest weekly or continuously? Why?

**Condense the Following**

26) $\log_5 2+\log_5 3+\log_5 4$

27) $\log_2 48-\frac{1}{3}\log_2 27$

28) $\frac{2}{3}\ln 8-2\ln 5$

29) $\log M-3\log N$

30) $\frac{1}{2}\left(\log M-\log N-\log P\right)$

31) $5\left(\log A+\log B\right)-2\log C$

32) $\log_8\sqrt{80}-\log_8\sqrt{5}$

33) $\frac{1}{2}\ln 25+\ln 2$

**Expand the following:**

34) $\ln\frac{1}{\sqrt{t}}$

35) $\log_3 11x$

36) $\log_3\sqrt[3]{x+1}$

37) $\log_4\sqrt{3x}$

38) $\log_2\frac{z}{17}$

39) $\ln\frac{5}{x-2}$

40) $\log_5\sqrt{xy}$

41) $\ln\frac{x^2y}{z^7}$

**Solve:**

42) $2\log_6 4-\frac{1}{4}\log_6 16=\log_6 x$

43) $\log_6 18+\log_6\left(x-2\right)=2$

44) $\log x+\log x+\log x=\log 8$

45) $\log\left(x-3\right)-\log\left(x+1\right)=\log 8$

46) $3\log_7 4+4\log_7 3=\log_7 x$

47) $\log_5\left(x+2\right)-\log_5\left(x-2\right)=1$

48) $\log_2\left(4x+10\right)-\log_2\left(x+1\right)=4$

49) $\log_6 x=\frac{1}{2}\log_6 9+\frac{1}{3}\log_6 27$

50) $\log 5+\log x=1$

51) $\log 5+\log x^2=2$

52) $\log x-\log 4=1$

53) $\log 3-\log y=2$

54) $\log_3\left(2x-5\right)-\log_3\left(x^2+4x+4\right)=-2$

**Evaluate the following**

55) $\log_2 32$

56) $\log_9 27$

57) $\ln e$

58) $\log_{10}(0.001)$

59) $\log_6 1$

60) $\log_7\sqrt[3]{\frac{1}{7}}$

61) $\ln\frac{1}{e}$

62) $\log_{\frac{1}{3}} 9$ 63) $\log_4 \sqrt[5]{4}$ 64) $\ln 1$ 65) $\log_{36} 6$ 66) $\log_5 \frac{1}{125}$ 67) $\log_4 16$ 68) $\ln \frac{1}{e^3}$

69) $\log_7 7$ 70) $\log_{16} 2$ 71) $\log_3 \sqrt{\frac{1}{9}}$ 72) $\log_4 x = 3$ 73) $\log_9 x = \frac{1}{2}$ 74) $\log_x 8 = 3$

**Solve**

75) $2^x = 45$ 76) $3^x = 3.6$ 77) $10^{2y} = 52$ 78) $7^{3y} = 126$ 79) $3^{x+4} = 6$

80) $10^{x+6} = 250$ 81) $3e^x = 42$ 82) $\frac{1}{4}e^x = 5$ 83) $\frac{1}{2}e^{3x} = 20$ 84) $250(1.04)^x = 1000$

85) $300e^{\frac{x}{2}} = 9000$ 86) $1000^{0.12x} = 25000$ 87) $\frac{1}{5}\left(4^{x+2}\right) = 300$ 14) $6 + 2^{x-1} = 1$ 88) $7 + e^{2-x} = 28$

89) $8 - 12e^{-x} = 7$ 90) $4 + e^{2x} = 10$ 91) $32 + e^{7x} = 46$ 92) $23 - 5e^{x+1} = 3$ 93) $4\left(1 + e^{\frac{x}{3}}\right) = 84$

**VI. Word Problems**

94) Find the amount of money that results if \$100 is invested at 4% compounded quarterly for 2 years.

95) A sum of \$1500 was invested for 5 years, and the interest was compounded monthly. If this sum amounted to \$1633 in the given time, what was the interest rate? compounded quarterly after a period of 2 years.

96) How many years will it take for an initial investment of \$25,000 to grow to \$80,000? Assume a rate of interest of 7% compounded continuously.

97) The size of P of a certain insect population at time t (in days) obeys the equation $P = 500e^{0.02t}$. After how many days will the population reach 1000? When will it reach 2000?

98) The half-life of radium is 1690 years. If 28 grams are present now, how much will there be in 25 years?

99) The population of a southern city follows an exponential model If the population doubled in size over an 18 month period and the current population is 10,000, what will the population be 2 years from now?

100) Salt (NaCl) decomposes in water into sodium and chloride ions according to the law of uninhibited decay. If the initial amount of salt is 25 kilograms and after 10 hours, 15 kg. of salt is left, how much salt is left after 1 day?