AFM Unit 3 Exp and Log Test – Study Guide

***Complete #1-25 all. For #26-100, pick and choose questions to do as needed.**

I. Write the equation in exponential form. Do NOT evaluate.

1. $\log_5 47 = x$ $5^x = 47$ 2. $\ln 9 = z$ $e^z = 9$

II. Write the equation in logarithmic form. Do NOT evaluate.

3. $7^3 = x$ $\log_7 x = 3$ 4. $e^{5x} = k$ $\ln K = 5x$

III. Evaluate each expression. Write your answer in the blank space. Round your answer to 2 decimal places.

5. $\log_4 \sqrt{36}$ $4^x = \sqrt{36}$ $x = \frac{\log \sqrt{36}}{\log 4}$ 6. $\log_3 25 = x$ $3^x = 25$ $x = \frac{\log 25}{\log 3}$ 7. ~~$5^{\log_5 30}$~~

5. $x = 1.29$ 6. $x = 2.93$ 7. 30

8. $\ln 21$ on calc 9. $\log_4 1 = x$ $4^x = 1$ 10. Which is larger, $\log_3 300$ or $\log_6 800$? $3^x = 300$ 5.19 $6^x = 800$ 3.73

8. 3.04 9. 0 10. _____

IV. Solve each equation. Show your work. Round your answer to 2 decimal places.

11. $5^{4x+6} = 23$ $4x+6 = \frac{\log 23}{\log 5}$ 12. $e^{.26t} = 68$ $\ln e^{.26t} = \ln 68$ $.26t = \ln 68$ 13. $20e^{-.4t} = 160$ $\ln e^{-.4t} = \ln 8$ $-.4t = \ln 8$

11. $x = -1.01$ 12. $t = 16.23$ 13. $t = -5.20$

14. $3^{6x-3} = 6$ $6x - 3 = \frac{\log 6}{\log 3}$ 15. $\log (t - 20) = 6$ $10^{\log(t-20)} = 10^6$ $t - 20 = 1000020$ $+20 \quad +20$ 16. $\frac{3}{3} \ln (2 - x) = \frac{9}{3}$ $e^{\ln(2-x)} = e^3$ $2 - x = e^3$

14. $x = 0.77$ 15. $t = 1000020$ 16. $x = -18.09$

VI. Solve the following problems. Be sure to first include the formula you will use for the problem, then show all of your work! Round your answer to 2 decimal places. Please circle or box your final answer!

17. The number of a certain species of fish is modeled by the function $n(t) = 23e^{0.053t}$ where t is measured in years and n(t) is measured in millions.
    a) Express as a percentage, the relative rate of growth of the fish population. 5.3%.
    b) What was the initial fish population? 23 million
    c) What will the fish population be after 7 years? Show your work! 33.33 million

18. Ashley will be buying a car for $24000 in five years. How much money should she ask her parents for now so that, if she invests at 8.2% compounded continuously, she will have enough to buy the new car?

$24000 = Pe^{.082(5)}$

$P = \$15927.61$

19. Suppose that $6,045 is invested in a savings account paying 7.25% interest per year.

a) Write the formula for the amount in the account after t years if interest is compounded semiannually. $A = 6045\left(1 + \frac{.0725}{2}\right)^{2t}$

b) Find the amount in the account after 5 years. $8630.61

c) How long will it take for the amount in the account to grow to $9,500?

$9500 = 6045\left(1 + \frac{.0725}{2}\right)^{2t}$ $t = 6.33$ yrs

20. A sum of $1475 was invested for six years and the interest was compounded monthly. If this sum amounted to $4832.46 after the given time, what was the interest rate? Show your work!

$4832.46 = 1475\left(1 + \frac{r}{12}\right)^{12 \cdot 6}$

$(3.28)^{1/72} = \left[\left(1 + \frac{r}{12}\right)^{72}\right]^{1/72}$

$3.28^{1/72} - 1 = \left(\frac{r}{12}\right)^{12}$

$r = .1996 \rightarrow 19.96\%$

21. Find a function that models the rabbit population in a North Carolina county after 1996. Assume that the population grows exponentially. In 1996, the population was 20,000 rabbits and in 2000 the population was 53,000. In what year will the population reach 100,000?

$53000 = 20000e^{4r}$

$r = .0.2425$

$100000 = 20000e^{.2425t}$

$t = 6.68$

22. The half life of radium-226 is 5 days. After 25 days a sample has been reduced to 0.375 g.

a) What was the initial mass of the sample? $r = \frac{\ln 2}{5} = .1386$

$.375 = m_0 e^{-.1386(25)}$

$m_0 = 11.99\,g$

b) After how many days will only 0.15 g remain?

$0.15 = 11.99e^{-.1386t}$

$0.0125 = e^{-.1386t}$

$\ln(.0125) = -.1386t$

$t = 31.6$ days

23. What interest rate is required for an investment with continuously compounded interest to double in 20 years?

$2 = e^{r(20)}$

$r = 0.0347$ 3.47%

24. How long does it take for an investment to double in value if it is invested at 5% compounded weekly (n=52)?

$2 = \left(1 + \frac{.05}{52}\right)^{52t}$

$t = 13.34$

25. If the interest rates are the same, would you choose a savings account that compounded interest weekly or continuously? Why?

You would earn more money faster

Condense the Following

26) $\log_5 2+\log_5 3+\log_5 4$

$\log_5(2\cdot 3\cdot 4)$

27) $\log_2 48-\frac{1}{3}\log_2 27$

$\log_2 \frac{48}{27^{1/3}}$ or $\log_2 \frac{48}{\sqrt{27}}$

28) $\frac{2}{3}\ln 8-2\ln 5$

$\ln \frac{8^{2/3}}{5^2}$

29) $\log M-3\log N$

$\log \frac{M}{N^3}$

30) $\frac{1}{2}(\log M-\log N-\log P)$

$\log\left(\frac{M/N}{P}\right)^{1/2}$

31) $5(\log A+\log B)-2\log C$

$\log \frac{(AB)^5}{C^2}$

32) $\log_8\sqrt{80}-\log_8\sqrt{5}$

$\log_8 \frac{\sqrt{80}}{\sqrt{5}}$

33) $\frac{1}{2}\ln 25+\ln 2$

$\ln(25^{1/2}\cdot 2)$

OR $\ln 2\sqrt{25}$

Expand the following:

34) $\ln\frac{1}{\sqrt{t}}$

$\ln 1-\frac{1}{2}\ln t$

35) $\log_3 11x$

$\log_3 11+\log_3 x$

36) $\log_3\sqrt[3]{x+1}$

$\frac{1}{3}(\log_3(x+1))$

37) $\log_4\sqrt{3x}$

$\frac{1}{2}(\log_4 3+\log_4 x)$

38) $\log_2\frac{z}{17}$

$\log_2 z-\log_2 17$

39) $\ln\frac{5}{x-2}$

$\ln 5-\ln(x-2)$

40) $\log_5\sqrt{xy}$

$\frac{1}{2}(\log_5 x+\log_5 y)$

41) $\ln\frac{x^2y}{z^7}$

$2\ln x+\ln y-7\ln z$

Solve:

42) $2\log_6 4-\frac{1}{4}\log_6 16=\log_6 x$

$\frac{4^2}{16^{1/4}}=x$

$x=8$

43) $\log_6 18+\log_6(x-2)=2$

$18(x-2)=6^2$

$18x-36=36$

$x=4$

44) $\log x+\log x+\log x=\log 8$

$x^3=8$

$x=2$

45) $\log(x-3)-\log(x+1)=\log 8$

$\frac{x-3}{x+1}=8$

$x-3=8x+8$ $\quad x=\frac{-11}{7}$

46) $3\log_7 4+4\log_7 3=\log_7 x$

$4^3\cdot 3^4=x$

$x=5184$

47) $\log_5(x+2)-\log_5(x-2)=1$

$\frac{x+2}{x-2}=5^1$

$x+2=5x-10$ $\quad x=3$

48) $\log_2(4x+10)-\log_2(x+1)=4$

$\frac{4x+10}{x+1}=2^4$

$4x+10=16(x+1)$ $\quad x=-\frac{1}{2}$

$4x+10=16x+16$

49) $\log_6 x=\frac{1}{2}\log_6 9+\frac{1}{3}\log_6 27$

$x=9^{1/2}\cdot 27^{1/3}$

$x=9$

50) $\log 5+\log x=1$

$5x=10^1$

$5x=10$ $\quad x=2$

51) $\log 5+\log x^2=2$

$5x^2=10^2$

$5x^2=100$ $\quad x=\pm\sqrt{20}$

$x^2=20$

52) $\log x-\log 4=1$

$\frac{x}{4}=10^1$

$x=40$

53) $\log 3-\log y=2$

$\frac{3}{y}=10^2$ $\quad \frac{3}{y}=100$ $\quad y=\frac{3}{100}$

54) $\log_3(2x-5)-\log_3(x^2+4x+4)=-2$

$\frac{2x-5}{x^2+4x+4}=3^{-2}$ $\quad \frac{2x-5}{x^2+4x+4}=\frac{1}{9}$

$x^2+4x+4=18x-45$ $\quad x^2-14x+49=0$

$(x-7)(x-7)=0$

$x=7$

Evaluate the following

55) $\log_2 32$

$2^x=32$

$x=5$

56) $\log_9 27$

$9^x=27$

$x=3/2$

57) $\ln e$

1

58) $\log_{10}(0.001)$

$10^x=\frac{1}{1000}$

$x=-3$

59) $\log_6 1$

0

60) $\log_7\sqrt[3]{\frac{1}{7}}$

$7^x=\left(\frac{1}{7}\right)^{1/3}$

$x=-1/3$

61) $\ln\frac{1}{e}$

$\ln e^{-1}$

$x=-1$

62) $\log_{\frac{1}{3}} 9$ $\left(\frac{1}{3}\right)^x = 9$, $x = -2$

63) $\log_4 \sqrt[3]{4}$ $4^x = 4^{1/5}$, $x = 1/5$

64) $\ln 1$ 0

65) $\log_{36} 6$ $36^x = 6$, $x = 1/2$

66) $\log_5 \frac{1}{125}$ $5^x = \frac{1}{125}$, $x = -3$

67) $\log_4 16$ $4^x = 16$, $x = 2$

68) $\ln \frac{1}{e^3}$ $\ln e^{-3}$, $x = -3$

69) $\log_7 7$ 1

70) $\log_{16} 2$ $16^x = 2$, $x = \frac{1}{4}$

71) $\log_3 \sqrt{\frac{1}{9}}$ $3^x = \frac{1}{9}^{1/2}$, $x = -1$

72) $\log_4 x = 3$ $4^3 = x$, $x = 64$

73) $\log_9 x = \frac{1}{2}$ $9^{1/2} = x$, $x = 3$

74) $\log_x 8 = 3$ $x^3 = 8$, $x = 2$

Solve

75) $2^x = 45$ $x = \frac{\log 45}{\log 2}$, $x = 5.49$

76) $3^x = 3.6$ $x = \frac{\log 3.6}{\log 3}$, $x = 1.17$

77) $10^{2y} = 52$ $2y = \frac{\log 52}{\log 10}$, $2y = 1.72$, $y = 0.86$

78) $7^{3y} = 126$ $3y = \frac{\log 126}{\log 7}$, $3y = 0.83$

79) $3^{x+4} = 6$ $x+4 = \frac{\log 6}{\log 3}$, $x = -2.37$

80) $10^{x+6} = 250$ $x+6 = \frac{\log 250}{\log 10}$, $x = -3.6$

81) $3e^x = 42$ $e^x = 14$, $x = \ln(14)$, $x = 2.64$

82) $\frac{1}{4}e^x = 5$ $e^x = 20$, $x = \ln(20)$, $x = 2.995$

83) $\frac{1}{2}e^{3x} = 20$ $e^{3x} = 40$, $3x = \ln(40)$, $x = 1.23$

84) $250(1.04)^x = 1000$ $1.04^x = 4$, $x = \frac{\log 4}{\log 1.04}$, $x = 35.35$

85) $300e^{\frac{x}{2}} = 9000$ $e^{x/2} = 30$, $\frac{x}{2} = \ln(30)$, $x = 6.8$

86) $1000^{0.12x} = 25000$ $.12x = \frac{\log 25000}{\log 1000}$, $x = 12.25$

87) $\frac{1}{5}\left(4^{x+2}\right) = 300$ 14) $4^{x+2} = 1500$, $x+2 = \frac{\log 1500}{\log 4}$, $x = 3.28$

$6 + 2^{x-1} = 1$ $2^{x-1} = -5$, No solu

88) $7 + e^{2-x} = 28$ $e^{2-x} = 21$, $2 - x = \ln(21)$, $x = -1.04$

89) $8 - 12e^{-x} = 7$ $-12e^{-x} = -1$, $e^{-x} = 1/12$, $-x = \ln(1/12)$, $x = 2.48$

90) $4 + e^{2x} = 10$ $e^{2x} = 6$, $2x = \ln(6)$, $x = 0.90$

91) $32 + e^{7x} = 46$ $e^{7x} = 14$, $7x = \ln(14)$, $x = 0.38$

92) $23 - 5e^{x+1} = 3$ $-5e^{x+1} = -20$, $e^{x+1} = 4$, $x+1 = \ln(4)$, $x = 0.39$

93) $4\left(1 + e^{\frac{x}{3}}\right) = 84$ $1 + e^{\frac{x}{3}} = 21$, $e^{x/3} = 20$, $\frac{x}{3} = \ln(20)$, $x = 8.987$

VI. Word Problems

94) Find the amount of money that results if $100 is invested at 4% compounded quarterly for 2 years.

$A(t) = 100\left(1 + \frac{.04}{4}\right)^{4 \cdot 2}$ $A(t) = \$108.29$

95) A sum of $1500 was invested for 5 years, and the interest was compounded monthly. If this sum amounted to $1633 in the given time, what was the interest rate? ~~compounded quarterly after a period of 2 years~~

$\frac{1633}{1500} = \frac{1500\left(1 + \frac{r}{12}\right)^{12(2)}}{1500}$ $1.08 = \left(1 + r/12\right)^{24}$, $1.08^{1/24} = 1 + r/12$, $r = 0.039$, 3.9%

96) How many years will it take for an initial investment of $25,000 to grow to $80,000? Assume a rate of interest of 7% compounded continuously.

$80000 = 25000e^{.07t}$, $3.2 = e^{.07t}$, $\ln 3.2 = .07t$, $t = 16.62$ yrs

97) The size of P of a certain insect population at time t (in days) obeys the equation $P = 500e^{0.02t}$. After how many days will the population reach 1000? When will it reach 2000?

$1000 = 500e^{.02t}$, $t = 34.66$ yrs

$2000 = 500e^{.02t}$, $t = 69.31$ yrs

98) The half-life of radium is 1690 years. If 28 grams are present now, how much will there be in 25 years?

$r = \frac{\ln 2}{1690}$ $m(t) = 28e^{-\ln 2/1690 \cdot 25/1690}$ $m(t) = 27.9998$ g

99) The population of a southern city follows an exponential model If the population doubled in size over an 18 month period and the current population is 10,000, what will the population be 2 years from now?

$2 = e^{r(1.5)}$, $r = 0.46$ $n(t) = 10000e^{.46(2)}$, $= 25092$

100) Salt (NaCl) decomposes in water into sodium and chloride ions according to the law of uninhibited decay. If the initial amount of salt is 25 kilograms and after 10 hours, 15 kg. of salt is left, how much salt is left after 1 day?