Unit 5 Matrices Review

1. State the 4 x 4 identity matrix.
2. Explain why a 3 x 2 matric does not have an inverse.
3. True or false:

$$\begin{bmatrix}6 & 1\\ 0 & 4\end{bmatrix}\cdot\begin{bmatrix}\frac{1}{6} & -\frac{1}{24}\\ 0 & \frac{1}{4}\end{bmatrix}=I$$

1. $\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$

It is not square.

yes, $=\begin{bmatrix}1&0\\0&1\end{bmatrix}$

Use the matrices below to answer questions 4-11

$$R=\begin{bmatrix}9 & 6 & 7\\ 2 & 5 & 0\\ 10 & 3 & 11\end{bmatrix}\quad S=\begin{bmatrix}5 & -2 & 4 & 3\\ 0 & 8 & 0 & -1\end{bmatrix}\quad T=\begin{bmatrix}8 & -1 & 6\\ -7 & 0 & 2\\ 4 & 9 & -5\end{bmatrix}\quad U=\begin{bmatrix}2 & x & -2 & 11\\ -4 & 3 & 5 & 9\end{bmatrix}$$

4. What are the dimensions of matrix S? 2 x 4
5. Identify the element r_{23}. 0
6. The value of x is what element? U_{12}
7. Which matrices have the same dimensions? R & T, S & U
8. R + U = Not possible
9. -0.5R= $\begin{bmatrix}-4.5 & -3 & -3.5\\ -1 & 2.5 & 0\\ -5 & -1.5 & -5.5\end{bmatrix}$
10. U - S = $\begin{bmatrix}-3 & -2-x & 6 & 8\\ -4 & -5 & 5 & 10\end{bmatrix}$
11. 3R + T = $\begin{bmatrix}27 & 18 & 21\\ 6 & 15 & 0\\ 30 & 9 & 33\end{bmatrix}+\begin{bmatrix}8 & -1 & 6\\ -7 & 0 & 2\\ 4 & 9 & -5\end{bmatrix}=\begin{bmatrix}35 & 17 & 27\\ -1 & 15 & 2\\ 34 & 18 & 28\end{bmatrix}$

Perform the indicated operations without a calculator.

12. $\begin{bmatrix}1 & 3 & 3\\ -2 & 4 & 4\end{bmatrix}-2\begin{bmatrix}5 & 5 & -2\\ 3 & 6 & 2\end{bmatrix}$

$\begin{bmatrix}1 & 3 & 3\\ -2 & 4 & 4\end{bmatrix}-\begin{bmatrix}10 & 10 & -4\\ 6 & 12 & 4\end{bmatrix}$

$\begin{bmatrix}-9 & -7 & 7\\ -8 & -8 & 0\end{bmatrix}$

13. $\begin{bmatrix}6 & 1\\ 0 & 8\end{bmatrix}\cdot\begin{bmatrix}-4 & 3\\ 7 & 11\\ 3 & 1\end{bmatrix}$ (2x2 · 3x2)

Not possible

14. $\begin{bmatrix}-2 & 1\\ 4 & 0\\ 2 & 2\end{bmatrix}\cdot\begin{bmatrix}1\\5\end{bmatrix}$ 3×2 2×1 $\begin{bmatrix}-2+5\\4+0\\2+10\end{bmatrix}=\begin{bmatrix}-3\\4\\12\end{bmatrix}$

15. $\begin{vmatrix}1 & 0 & 2\\ -1 & 0 & 1\\ -1 & -2 & 0\end{vmatrix}$ 1 0 / -1 0 / -1 -2

(1)(0)(0) + (0)(1)(-1) + (2)(-1)(-2) = 4

(0)(-1)(0) + (1)(1)(-2) + (2)(0)(-1) = -2

4 - -2 = 6

16. $\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 7 & 8 & 9\end{bmatrix}+\begin{bmatrix}-1 & -2 & -3\\ -4 & -5 & -6\\ -7 & -8 & -9\end{bmatrix}=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$

17. If $E=\begin{bmatrix}2 & -1 & 3\\ 0 & 4 & 1\\ -6 & -2 & 4\end{bmatrix}$, find E^{-1}

$\begin{bmatrix}3/19 & -1/57 & -13/114\\ -1/19 & 13/57 & -1/57\\ 4/19 & 5/57 & 4/57\end{bmatrix}$

18. Solve the following system using the method of your choice (Cramer's or Gaussian). Do not use a calculator.

$z = -3x + 3y + 11$ → $3x - 3y + z = 11$

$-3x + 7y - 7z = 3$

$-2x + 2y - 6z = 30$

$D=\begin{vmatrix}3 & -3 & 1\\ -3 & 7 & -7\\ -2 & 2 & -6\end{vmatrix}=-64$ $D_x=\begin{vmatrix}11 & -3 & 1\\ 3 & 7 & -7\\ 30 & 2 & -6\end{vmatrix}=64$

$D_y=\begin{vmatrix}3 & 11 & 1\\ -3 & 3 & -7\\ -2 & 30 & -6\end{vmatrix}=448$ $D_z=\begin{vmatrix}3 & -3 & 11\\ -3 & 7 & 3\\ -2 & 2 & 30\end{vmatrix}=448$

$x=\frac{64}{-64}=-1$ $y=\frac{448}{-64}=-7$ $z=\frac{448}{-64}=-7$

19. Solve the following systems using the indicated method.

a. Elimination

$(-5x-5y=25)2$

$(-2x-4y=16)-5$

$-10x-10y=50$

$10x+20y=-80$

$10y=-30$

$y=-3$

$-2x-4(-3)=16$

$x=-2$

b. Substitution

$x-3y=5$ $x=5+3y$

$-3x+6y=8$

$-3(5+3y)+6y=8$

$-15-9y+6y=8$

$-15-3y=8$

$-3y=23$

$y=-23/3$ $x=-18$

$x=5+3(-23/3)=-18$

c. You pick!

$-4x-6z=-12$

$-6x-4y-2z=6$

$-x+2y+z=9$

$\begin{bmatrix}-1 & 0 & -6\\ -6 & -4 & -2\\ -1 & 2 & 1\end{bmatrix}\begin{bmatrix}-12\\6\\9\end{bmatrix}$

$(-3, 7/4, 5/2)$

d. Gaussian Any!

$-6x-y+z=-7$

$4z=-6$

$4x-24y+24z=17$

$(5/4, -2, -3/2)$

e. Cramer's

$3a+b=-c+7$

$a+3b-c=13$

$b=2a-1$

$(2, 3, -2)$

f. Gaussian or Cramer's

$13=3x-y$ → $3x-y+0z=13$

$14y-3x+2z=-3$ $-3x+14y+2z=-3$

$z=2x-4y$ $2x-4y-z=0$

$(75/19, -22/19, 238/19)$

20. Solve the following system using your calculator.

$\begin{cases}3x+3y=19+z\\ 5x+4y-28=2z\\ 2(x+y)-12=z\end{cases}$ → $\begin{cases}3x+3y-z=19\\ 5x+4y-2z=28\\ 2x+2y-z=12\end{cases}$ $(4, 3, 2)$

Solve for the variables

21. $\begin{bmatrix}4x\\6y\end{bmatrix}+\begin{bmatrix}6y-6\\6x+12\end{bmatrix}=\begin{bmatrix}-14\\6\end{bmatrix}$

$4x+6y-6=-14$ → $4x+6y=-8$

$6y+6x+12=6$ → $6x+6y=-6$

$-(6x+6y=-6)$

$-6x-6y=6$

$-2x=-2$

$x=1$

$4(1)+6y=8$

$6y=4$

$y=2/3$

22. $\begin{vmatrix} 3 & -1 \\ 4 & x \end{vmatrix} = 25$ $3(x) - (-1)(4) = 25$ $3x + 4 = 25$ $3x = 21$ $x = 7$

23. $\begin{bmatrix} x & 2 \\ -1 & 5 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ y & 4 \end{bmatrix} = \begin{bmatrix} 9 & 2 \\ -3 & 22 \end{bmatrix}$ $x = 3$ $y = 0$

$\begin{bmatrix} 3x+2y & -2x+8 \\ -3+5y & 2+20 \end{bmatrix} = \begin{bmatrix} 9 & 2 \\ -3 & 22 \end{bmatrix}$

24. $\begin{bmatrix} 1 & -3 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 22 \end{bmatrix}$

$A^{-1} = \frac{1}{4}\begin{bmatrix} -2 & 3 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -1/2 & 3/4 \\ -1/2 & 1/4 \end{bmatrix}\begin{bmatrix} 10 \\ 22 \end{bmatrix}$

$= \begin{bmatrix} 23/2 \\ 1/2 \end{bmatrix} = \begin{bmatrix} 11.5 \\ .5 \end{bmatrix}$

25. $\begin{bmatrix} 2z - x & 4w + 5 \\ 2y - w & 10 \end{bmatrix} = \begin{bmatrix} y & 17 \\ 15 & 6z \end{bmatrix}$

$w = 3$ $y = 9$

$x = -17/3$ $z = 10/6 = 5/3$

26. The first number multiplied by 2 is the opposite of the second number. The third number is subtracted from the product of the second number and 3 to get 20. The sum of the first and third numbers is -5. Use a matrix equation to solve for these three numbers.

$2x = -y \rightarrow 2x + y + 0z = 0$

$3y - z = 20 \rightarrow 0x + 3y - z = 20$

$x + 0z = -5 \quad x + 0y + z = -5$

-3, 6, -2

27. Ugh! The theater department wants to be able to buy an Audrey II for Little Shop of Horrors next year, and to do so, it actually need $20,000 total revenue from the play. They decide to keep it open for a 4th night, at a far cheaper price so they can make it to their goal. Adult tickets will still be twice as much as student tickets. The sales for the 4th night are exactly like the second night. Set up and solve a matrix equation that shows how much should the theater department charge for each type of ticket to hit their target revenue? Adult = x Student = y 20000

$x = 2y$

SKIP

28. You make a VHS tape of your three favorite TV shows for your friend: Family Guy (x), Lost (y), and One Tree Hill (z). You can completely fill the tape with 7 episodes. You want include twice as many episodes of Lost as Family Guy. An episode of Family Guy lasts 30 minutes. An episode of Lost and One Tree Hills lasts 60 minutes. Your VCR tape can only hold 360 minutes of recording. How many episodes of each show can you tape?

$x + y + z = 7$

$y = 2x \rightarrow -2x + y + 0z = 0$

$30x + 60y + 60z = 360$

2 – Family Guy

4 – Lost

1 – one tree hill

29. Mrs. Stevens bought 15 tickets to a spaghetti supper and spent a total of $72. She bought adult tickets for $6 each and child tickets $4 each. How many of each kind of ticket did she buy?

$x + y = 15$

$6x + 4y = 72$

6 adult

9 child

30. A florist is making 5 identical bridesmaid bouquets for a wedding. She has $610 to spend (including tax) and wants 24 flowers for each bouquet. Roses (x) cost $6 each, tulips (y) cost $4 each, and lilies (z) cost $3 each. She wants to have twice as many roses as the other 2 flowers combined in each bouquet. How many roses, tulips, and lilies are in each bouquet? 610/5 = $122 each

$x = 2(y + z) \rightarrow x - 2y - 2z = 0$

$x + y + z = 24$

$6x + 4y + 3z = 122$

16 roses

2 tulips

6 lilies