Unit 5: Matrices Name: ____________________

Objectives:
CCSS.MATH.CONTENT.HSN.VM.C.6: Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
CCSS.MATH.CONTENT.HSN.VM.C.7: Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
CCSS.MATH.CONTENT.HSN.VM.C.8: Add, subtract, and multiply matrices of appropriate dimensions.
CCSS.MATH.CONTENT.HSN.VM.C.9: Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
CCSS.MATH.CONTENT.HSN.VM.C.10: Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
CCSS.MATH.CONTENT.HSN.VM.C.1: Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
CCSS.MATH.CONTENT.HSN.VM.C.12: Work with 2×2 matrices as a transformations of the plane, and interpret the absolute value of the determinant in terms of area.

Day	Lesson	Activity
1	Intro to Matrices Adding and Subtracting Matrices	
2	Matrix Multiplication (no calculator)	
3	2x2 Matrices, Determinants and Inverses	
4	3x3 Matrices, Determinants and Inverses	
5	**Review/Quiz**	**Quiz (no calculator)**
6	Solving systems of 2 and 3 Variables By Hand	
7	Cramer's Rule and Systems Gaussian Elimination	
8	Review	
9	**Test**	**Test (calculator active and inactive)**

Unit 5 Day 1: Organizing Data into Matrices

A ___________ is a rectangular array of numbers arranged in rows by columns. Mountains of real world data may be quickly processed when arranged in this rectangular format.

Each element or _____________in a matrix has a specific location or _________, read as a "row by column" location.

Example: $\begin{matrix} \mathbf{3} & \mathbf{-2} & \mathbf{5} \\ \mathbf{4} & \mathbf{0} & \mathbf{1} \end{matrix}$ The entry in the 2^{nd} row, 3^{rd} column, is identified as ___________.

The size or _________ of a matrix is simply the number of rows by the numbers of columns:________ . Thus, the size of the matrix in the example above is 2 by 3 or 2 x 3.

* * * Two matrices are equal **if and only if** they are the same size **and** their corresponding matrix elements are identical or equivalent.

Consider the data sets below for Aaron's Service Center

Auto Parts

Store #	2010	2011
103	$143,000	$188,000
205	$217,000	$195,000
135	$93,000	$135,000

Mechanic Services

Store #	2010	2011
103	$245,000	$305,000
205	$486,000	$475,000
135	$204,000	$193,000

What is Aaron's total revenue (parts + service) for Store #103 in 2010?	What is Aaron's total revenue (parts + service) for Store #205 in 2011?	What is Aaron's total revenue (parts + service) for Store #103 in 2011?

Notice that order of the entries is important. Each store has a specific set of data.
This process is called ___________________________________.

Matrix addition/subtraction is the process of adding or subtracting corresponding entries.

Thus, Aaron's Service Center data can be translated (in thousands of dollars) into matrix form (numerical data is transferred into the matrix):

Matrix operations also obey the commutative and associative properties as well as the additive identity and additive inverse.

Examples:

1. a) convert the data set into a matrix
   b) What are the dimensions of the matrix?
   c) The entry n_{32} is ________________

2004 Men's Olympic Gymnastics Individual Medal Winners

Gymnast	Floor Exercise	Pommel Horse	Still Rings	Vault	Parallel Bars	Horizontal Bars
Paul Hamm	9.725	9.700	9.587	9.137	9.837	9.837
Dae Eun Kim	9.650	9.537	9.712	9.412	9.775	9.725
Tae-Young Yang	9.512	9.650	9.725	9.700	9.712	0.475

2. The data represents a manufacturer's shipment records
   a) Display the data in matrix form, with columns representing years.
   b) What a_{23}? __________
   c) How many CDs were sold over the 6-year period?
   d) How many CDs and DVDs were sold in 2001?

Type	**1998**	**1999**	**2000**	**2001**	**2002**	**2003**
CD	847.0	938.9	942.5	881.9	803.3	745.9
DVD	0.5	2.5	3.3	7.9	10.7	17.5

3. Solve each equation for the variable (corresponding elements are equal)

$$\begin{bmatrix} 3x+2 & 5 & 2a \\ -18 & 6 & 5y+3 \end{bmatrix} = \begin{bmatrix} 8 & 2n-10 & 0 \\ c & 3k & -7 \end{bmatrix}$$

Unit 5 Day 1 Practice: Intro to Matrices

Write the dimensions of each matrix. Identify the indicated element.

1. $\begin{bmatrix} 2 \\ -3 \\ -6 \end{bmatrix}; a_{21}$

2. $\begin{bmatrix} 5 & -7 & 23 & 10 \\ -9 & 3 & 5 & -2 \\ 1 & 9 & 0 & 2 \end{bmatrix}; a_{23}$

6. $\begin{bmatrix} 5 & 8 & -7 & -4 \end{bmatrix}; a_{14}$

Use the table for Exercises 7–10.

7. Display the data in a matrix with the types of unemployment in the columns.

8. State the dimensions of the matrix.

9. Identify a_{21}, and tell what it represents.

10. Identify a_{16}, and tell what it represents.

Unemployment by Category

	June, 1992	**June, 1996**
Construction	17.6%	9.5%
Manufacturing	8.3%	5.1%
Transportation	5.4%	4.5%
Sales	8.7%	6.4%
Finance	4.0%	2.6%
Services	6.6%	5.1%
Government	3.5%	2.7%

Source: *U.S. News & World Report*

Unit 5 Day 1: Adding and Subtracting Matrices

Plain and simple, matrices are rectangular arrangements of numbers. These numbers may be data or they may be numbers from an equation. Today, matrices are used in many areas of work and research. Some of the many areas include electrical circuits, battery power outputs, and 2-dimensional and 3-dimensional screen images that create realistic movements. In geology matrices are used in seismic surveys. Real world data like banking, profit and loss, movie ticket sales, and on a big level, gross domestic profit! The base element for robot movement is matrices.

Vocabulary

Matrix –

Element –

Dimensions –

ROW X COLUMN – R X C –

Rules for Matrix Addition/Subtraction:

1.

2.

3.

Examples:

1. $\begin{bmatrix} 2 & -4 & 3 \\ 0 & 1 & 12 \end{bmatrix} + \begin{bmatrix} 0 & 7 & -2 \\ 13 & -1 & 5 \end{bmatrix}$

2. $2\begin{bmatrix} 6 & 5 \\ 10 & 0 \end{bmatrix} - \begin{bmatrix} -4 & 3 \\ 0 & -1 \end{bmatrix}$

Properties of Matrix Addition: If A, B, and C are m x n matrices (m rows and n columns) then:

1. A + B is an m x n matrix

2. A + B = B + A

3. (A + B) C = A + (B + C)

4. There is a unique matrix O such that O + A = A + O = A

5. For each A there exists a unique opposite, - A, A + (-A) = O

Matrix equations

Matrix equations are solved much like an equation in one variable, only the variable is an unknown matrix and the numbers are matrices!!

Examples:

1. $x - 6 = 13$

2. $X - \begin{bmatrix} 2 & 1 \\ 0 & -4 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 7 \\ 2 & 13 \\ -12 & 0 \end{bmatrix}$

3. $\begin{bmatrix} 2 & 1 \\ 0 & -3 \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ -10 & 2 \end{bmatrix} - X$

4. $2X + \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 7 \\ -5 \\ -1 \end{bmatrix}$

Finding the Value of a Variable

1. $\begin{bmatrix} a & 2b \\ c-2 & d+3 \end{bmatrix} = \begin{bmatrix} 5 & -7 \\ 10 & 10 \end{bmatrix}$

2. $\begin{bmatrix} 5 & 1 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} 2 & -13 \\ -10 & -10 \end{bmatrix} = \begin{bmatrix} 2x+1 & -4x \\ 5z & 2.5z - x \end{bmatrix}$

Day 1 Practice: Adding and Subtracting Matrices

Find the value of each variable.

1. $\begin{bmatrix} a & 2b \\ c-2 & d+3 \end{bmatrix} = \begin{bmatrix} 5 & -7 \\ 10 & 10 \end{bmatrix}$

3. $\begin{bmatrix} 5 & 1 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} 2 & -13 \\ -10 & -10 \end{bmatrix} = \begin{bmatrix} 2x+1 & -4x \\ 5z & 2.5z-x \end{bmatrix}$

Find each sum or difference.

6. $\begin{bmatrix} -1 & 2 \\ 3 & -1 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ -3 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 2 & 0 \end{bmatrix}$

7. $\begin{bmatrix} 8 & -5 & -5 \\ 4 & -10 & 10 \\ 2 & -15 & -15 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 1 \\ 1 & -2 & -2 \\ -2 & -3 & 3 \end{bmatrix}$

8. $\begin{bmatrix} -2 & -1 \\ -3 & 1 \\ -1 & -1 \end{bmatrix} - \begin{bmatrix} -2 & -2 \\ 3 & -1 \\ 0 & -2 \end{bmatrix} + \begin{bmatrix} -2 & 1 \\ 0 & 3 \\ -3 & -3 \end{bmatrix}$

9. $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} -1 \\ -3 \\ 5 \end{bmatrix} + \begin{bmatrix} -10 \\ -7 \\ 11 \end{bmatrix} - \begin{bmatrix} -3 \\ -5 \\ -6 \end{bmatrix}$

Solve each matrix equation.

10. $X - \begin{bmatrix} 3 & 4 \\ 4 & 2 \\ 1 & 9 \end{bmatrix} = \begin{bmatrix} 5 & 7 \\ 9 & 12 \\ 3 & 2 \end{bmatrix}$

11. $X + \begin{bmatrix} 20 & -9 & -3 \\ 19 & -2 & -5 \\ -1 & 0 & -8 \end{bmatrix} = \begin{bmatrix} -7 & 92 & -5 \\ 0 & 91 & -6 \\ -9 & -1 & 12 \end{bmatrix}$

12. $\begin{bmatrix} -2 & -3 \\ 2 & 2 \end{bmatrix} = X - \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix}$

Day 2: Matrix Multiplication
Scaler Multiplication:

Consider doubling the cost of movie tickets as shown below.

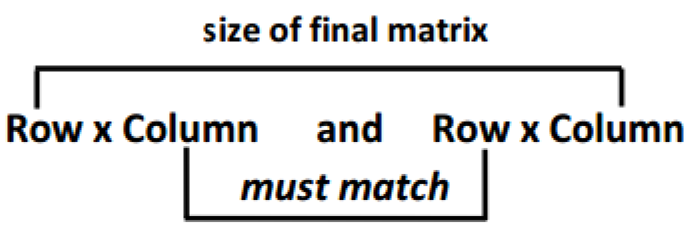
Children	$3.50		Children	$7.00
Adult	$8.50	**doubles to** →	Adult	$17.00
Sr. Citizens	$5.50		Sr. Citizens	$11.00

Mathematically, in matrix form:

Notice that every entry was doubled. In general, multiplying or dividing a matrix by a constant factor is called ____. The word "scaler" comes from the root word "scale."

Example: Simplify $-3 \times \begin{bmatrix} 3 & -2 & 5 \\ 4 & 0 & -1 \end{bmatrix} =$

Matrix multiplication is a bit more complicated.
To multiply two matrices the column size of the first matrix must match the row size of the second matrix. In other words:



To multiply two matrices:
1. Check for a "______________" and note the "_____________;" No match means no multiplication is possible

2. Multiply ________________ of left matrix with __________________ of right matrix. Now sum up all products. This becomes the element ______ in our matrix. For the element __________, repeat multiplication of ____________________ of left matrix with ________________________ of right matrix. Sum up products. Note the element address tells you that you will need to multiply the row 1 elements with the column 2 elements.

3. Repeat until all _____________________ of left matrix have been multiplied with all ________________ of the right matrix.

Example:

$$\begin{bmatrix} 3 & 2 & 0 \\ -1 & 4 & -2 \end{bmatrix} \times \begin{bmatrix} 1 & 4 & 0 \\ -2 & 3 & 2 \\ 1 & 0 & -3 \end{bmatrix}$$

Practice:

1. $\begin{bmatrix} 1 & 2 \\ -4 & 3 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 5 & 7 & 0 \\ -2 & 3 & 1 \end{bmatrix}$

2. $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 4 & -2 \\ 1 & 6 \end{bmatrix}$

3. $\begin{bmatrix} 3 & 2 & 0 \\ 1 & -1 & 5 \end{bmatrix} \cdot \begin{bmatrix} 4 & -2 & 3 \\ 2 & 0 & 4 \end{bmatrix}$

Unit 5 Day 2 Practice: Matrix Multiplication

Use matrices *A, B, C, D,* and *E* to find each product, sum, or difference, if possible. If not possible, write *product undefined, sum undefined,* or *difference undefined.*

$A = \begin{bmatrix} 1 & -1 \\ 3 & -2 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 2 \\ -2 & 1 \\ -1 & 0 \end{bmatrix}$ $C = \begin{bmatrix} 3 & -3 & -1 \\ 2 & -2 & 4 \end{bmatrix}$ $D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $E = \begin{bmatrix} 3 \\ -3 \\ 2 \end{bmatrix}$

1. $3AB$ **2.** $2A + 4D$ **3.** $5D - A$ **11.** DC **14.** $2D$ **15.** BE

16. $0.2B$ **17.** $\frac{1}{4}C$ **19.** DE

Find the dimensions of the product matrix. Then find each product.

21. $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} [1\ \ 2\ \ 3\ \ 4]$

22. $\begin{bmatrix} 1 & 2 & 12 \\ 12 & 2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 4 \\ 4 & 3 \\ 5 & 2 \end{bmatrix}$

Find each product if possible. If not possible, write *product undefined.*

24. $-12 \begin{bmatrix} -6 & -2 \\ -5 & -6 \\ 0 & 1 \end{bmatrix}$

25. $\begin{bmatrix} 3 & 2 \\ 4 & 6 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} -3 & 3 & -2 \\ -2 & 5 & -1 \end{bmatrix}$

26. $\begin{bmatrix} 0 & 1 & 0 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} -2 & 2 & 2 \\ -1 & 1 & 1 \\ 0 & -1 & -1 \end{bmatrix}$

Solve each equation. Check your answers.

30. $2\begin{bmatrix} 0 & 1 \\ 3 & -4 \end{bmatrix} - 3X = \begin{bmatrix} 9 & -6 \\ 1 & -2 \end{bmatrix}$

31. $\frac{1}{2}X + \begin{bmatrix} 5 & -1 \\ 0 & \frac{2}{3} \end{bmatrix} = 2\begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix}$

Unit 5 Day 3: 2x2 Matrices, Determinants and Inverses
Data transferred over the internet is encoded or encrypted so that someone attempting to illegally access the data will find something that is unintelligible. One way to encrypt messages and data uses matrices and their inverses. Also, inverse matrices are used to simply solve matrix equations.

The first step to calculating an inverse is to find the determinant. Using the determinant, we will be able to calculate the inverse.

Determinant – abbreviated **det** and symbol | |
For a 2×2 matrix, its determinant is found by ____:

Given a matrix A= $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, where a, b, c, and d are real numbers $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$

Det A =

Example: Compute the determinant

1. $A = \begin{bmatrix} -3 & 4 \\ 2 & -5 \end{bmatrix}$

2. $B = \begin{bmatrix} 8 & -7 \\ 2 & 3 \end{bmatrix}$

Matrix Inverse
What does an inverse do?
It undoes something. The additive inverse of x is ______________ and the multiplicative inverse of x is ____________ or ___________.

The inverses take us back to our identities: additive identity is _____ and the multiplicative identity is ____.

The ____________________ of a matrix and its inverse will equal "1." The matrix identity is called, the ________________________________; it is equivalent to "1" in matrix terminology. So, a matrix multiplied by _____________ is equal to the matrix.

The identity matrix of a 2x2 is

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Note: the identity matrix is identified with a capital I and a subscript indicating the dimensions. The matrix identity is ___________________. Regardless of the dimensions of the matrix, the identity matrix consists of a diagonal of ones and the corners are filled in with zeros.

Example: Multiply A by the identity matrix.

1. $\begin{bmatrix} 3 & 4 \\ -2 & 2 \end{bmatrix} \times$

Inverses: A number times its inverse (__________________) is equal to 1. A matrix ____________________ its inverse is also equal to "1."

Rule: When two matrices are multiplied, and the product is the identity matrix, we say the two matrices are ____________________.

Now we can find inverses and verify if a matrix is an inverse of another. It is a process, a pattern to follow.

Example: Is **B** the inverse of **A**?:

A=$\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ and B=$\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

If B is the inverse, then AB should equal the identity matrix.

The notation for the inverse of a matrix is __________________. In other words:

$$\boldsymbol{A\,A^{-1} = A^{-1}A = I}$$

(Remember I, identity matrix, is the "1" for matrices)

How do we find the inverse? We do scalar multiplication with the value of __________________ a mixed-up version of matrix A.

For matrix **A=** $\begin{bmatrix} \boldsymbol{a} & \boldsymbol{b} \\ \boldsymbol{c} & \boldsymbol{d} \end{bmatrix}$ **the inverse is:**

Note: det A is in the denominator. Determinants may not be equal to 0 as $\frac{1}{detA}$ would be undefined. So, a matrix with a determinant of 0 has no inverse and is called a ____________________.

Example: Find the inverse of the matrix,
1. Verify that $\det A \neq 0$
2. Set up inverse equation (note: switch a and d, and make c and b opposite sign)

$A = \begin{bmatrix} -2 & 2 \\ 5 & -4 \end{bmatrix}$	$\begin{bmatrix} 3 & 2 \\ 9 & 6 \end{bmatrix}$
$\begin{bmatrix} 6 & 3 \\ 5 & 8 \end{bmatrix}$	$\begin{bmatrix} 0.5 & 2.3 \\ 3 & 7.2 \end{bmatrix}$

Why do we need inverse matrices? Matrices are used for solving systems of simultaneous linear equations. Matrices are used for describing the quantum mechanics of atomic structure, designing computer game graphics, data encryption, analyzing relationships, and even plotting complicated dance steps!

If we are given a matrix equation such as : To solve a matrix equation $AX = B$, for x, you simply multiply each side by A^{-1}. This is the same process as solving an equation like $5x = 20$. To solve for x we multiply each side by 1/5, the inverse of 5.

To solve for a matrix:
1.$A^{-1}AX = A^{-1}B$, (why? A matrix times its inverse equals "1")
We are left with X on the left side which is what we are solving for. So: $X = A^{-1}B$

2. WARNING!!! You must always keep the order of the matrices uniform! A^{-1} B is NOT the same as B A^{-1}

Example: Solve the matrix equation for X

$$X = A^{-1}B$$

$$\begin{bmatrix} -2 & -5 \\ 1 & 3 \end{bmatrix} X = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

1. Multiply each side by the inverse. Remember: inverse times matrix=inverse times matrix.
ORDER COUNTS!!!

2. Multiplication by the inverse leaves X on the left side. Simplify the right side.

3. Calculate det A.

4. Using the determinant find A^{-1}

5. With A^{-1} simplify the right side of the equation to solve for X.

$$\begin{bmatrix} 3 & -4 \\ 4 & -5 \end{bmatrix} X = \begin{bmatrix} 0 & -22 \\ 0 & -28 \end{bmatrix}$$

Unit 5 Day 3 Practice: 2x2 Matrices, Determinants and Inverses

Find the matrix E^{-1} for each.

1. $E = \begin{bmatrix} 2 & -2 \\ -1 & 2 \end{bmatrix}$

2. $E = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

3. $E = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$

4. $E = \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$

5. $E = \begin{bmatrix} 1 & 4 \\ 1 & 3 \end{bmatrix}$

6. $E = \begin{bmatrix} 4 & 7 \\ 3 & 5 \end{bmatrix}$

Find the inverse of each matrix, if it exists. If it does not exist, write *no inverse* and explain why not.

7. $\begin{bmatrix} 3 & 4 \\ -3 & 4 \end{bmatrix}$

8. $\begin{bmatrix} 3 & 4 \\ 3 & 4 \end{bmatrix}$

9. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

10. $\begin{bmatrix} 30 & -4 \\ -25 & 3 \end{bmatrix}$

Solve each matrix equation.

11. $\begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix} X = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$

12. $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} X = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

13. $\begin{bmatrix} -2 & 3 \\ -4 & 5 \end{bmatrix} X = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$

Evaluate the determinant of each matrix.

14. $\begin{bmatrix} -3 & 4 \\ 1 & -1 \end{bmatrix}$

15. $\begin{bmatrix} 3 & 9 \\ 3 & 2 \end{bmatrix}$

16. $\begin{bmatrix} 1 & -4 \\ 2 & 6 \end{bmatrix}$

17. $\begin{bmatrix} 4 & -3 \\ 1 & -8 \end{bmatrix}$

18. $\begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$

19. $\begin{bmatrix} 1 & -12 \\ 3 & 0 \end{bmatrix}$

Determine whether the matrices are multiplicative inverses.

20. $\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}, \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$

21. $\begin{bmatrix} 4 & 9 \\ 2 & 6 \end{bmatrix}, \begin{bmatrix} 1 & -\frac{3}{2} \\ -\frac{1}{3} & \frac{2}{3} \end{bmatrix}$

22. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$

Unit 5 Day 4: 3x3 Matrices, Determinants and Inverses

The computations for 3×3 determinants are messier than for 2×2's. Various methods can be used, but the simplest method is probably the following:

1) Write down the determinant
2) Expand the determinant by rewriting ______________________________
3) Multiply along the down-to-the-right-diagonals, and then add them up. ↘
4) Multiply along the down-to-the-left-diagonals and then add these values up. ↙
5) Lastly subtract the down-left-diagonal total from the down to the right diagonal total

Example:

A= $\begin{bmatrix} 4 & -2 & 0 \\ -3 & 10 & 1 \\ 2 & 6 & -1 \end{bmatrix}$ det A= $\begin{vmatrix} 4 & -2 & 0 \\ -3 & 10 & 1 \\ 2 & 6 & -1 \end{vmatrix}$ expand:

Expand:	down to the right = down to the left = **Answer format:**

Find the determinant for the following matrices: (by hand)

1. $\begin{bmatrix} 1 & 4 & 0 \\ 2 & 3 & 5 \\ 0 & 1 & 0 \end{bmatrix}$

2. $\begin{bmatrix} 1 & 2 & 5 \\ 3 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}$

Calculator Method

Determinants: A 3x3 determinant may be calculated on a calculator. **YOU MUST BE ABLE TO CALCULATE THE DETERMINATE OF A 3x3 MATRIX BY HAND!**

Determinants:

1. Press **MATRIX** (2nd x^{-1}) > > to **EDIT**. Down to 1:[A]. **ENTER**
2. Enter the matrix dimensions: # rows **ENTER** # columns **ENTER**.
3. Enter the matrix data.
4. Press **2nd MODE** (QUIT)
5. Press **MATRIX** again. Go right once to **MATH**. Down to 1:**det**.
6. Press **MATRIX** again. Down to 1:[A]. **ENTER**. Answer is displayed.

Inverses:

1. Using a calculator, enter the data for your matrix
2. Now to calculate the inverse hit 2nd **MATRIX** select the matrix you want the inverse for and hit **ENTER**
3. Hit **x^{-1}**. The view screen will show: **$[A]^{-1}$**
4. **ENTER** the view screen will give the matrix inverse of your matrix.

Find the inverse of the matrices:

$\begin{bmatrix} -3 & 4 & 0 \\ 2 & -5 & 1 \\ 0 & 2 & 3 \end{bmatrix}$	$\begin{bmatrix} 3 & 4 & 1 \\ -2 & 0 & 2 \\ 1 & 5 & 3 \end{bmatrix}$

Solving a 3X3 Matrix Equation

Yes, we use the calculator.

1. Enter your matrix data into the calculator. Enter the 3x3 data into matrix A and the 3x1 data into matrix B.
2. Now to calculate the inverse hit 2nd **MATRIX** select the matrix you want the inverse for and hit **ENTER**
3. Hit $\mathbf{x}^{-1}$.The view screen will show: $[A]^{-1}$
4. With the matrix inverse on the screen hit (times) 2nd **MATRIX** [B] **ENTER**. You will see: $[A]^{-1}[B]$ hit **ENTER** one more time.
5. The resulting matrix will be our answer, the matrix that equals X.

***ORDER COUNTS! [B] [A]-1 WILL NOT WORK!!!

$\begin{bmatrix} 0 & 0 & 2 \\ 1 & 3 & -2 \\ 1 & -2 & 1 \end{bmatrix} X = \begin{bmatrix} 0 \\ -6 \\ 19 \end{bmatrix}$	Solve: $\begin{bmatrix} 2 & 1 & 3 \\ 5 & 1 & -2 \\ 1 & -1 & -9 \end{bmatrix} X = \begin{bmatrix} 1 \\ 8 \\ 5 \end{bmatrix}$

Unit 5 Day 4 Practice: 3x3 Matrices, Determinants and Inverses

Where necessary, use a graphing calculator. Find the inverse (A^{-1}) of each matrix, if it exists. If it does not exist, write *no inverse.*

1. $\begin{bmatrix} 1 & 2 & 0 \\ -2 & 0 & -3 \\ 3 & -1 & 5 \end{bmatrix}$ **2.** $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ 0 & 2 & 3 \end{bmatrix}$ **3.** $\begin{bmatrix} 2 & 4 & 3 \\ 0 & 5 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ **4.** $\begin{bmatrix} 0 & 2 & 0 \\ 2 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix}$

5. $\begin{bmatrix} 4 & 5 & 6 \\ 0 & 1 & 2 \\ 8 & 9 & 5 \end{bmatrix}$ **6.** $\begin{bmatrix} 1 & -1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **7.** $\begin{bmatrix} -1 & 0 & -1 \\ 0 & -2 & 0 \\ -2 & 0 & 3 \end{bmatrix}$ **8.** $\begin{bmatrix} -3 & -2 & -1 \\ 0 & 1 & 2 \\ 3 & 4 & -4 \end{bmatrix}$

Solve each equation for X.

9. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} X = \begin{bmatrix} 4 \\ -5 \\ 3 \end{bmatrix}$ **10.** $\begin{bmatrix} 1 & 2 & 0 \\ -2 & 0 & -3 \\ 3 & -1 & 5 \end{bmatrix} X = \begin{bmatrix} -1 \\ 12 \\ -20 \end{bmatrix}$ **11.** $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} X = \begin{bmatrix} 3 \\ 4 \\ 3 \end{bmatrix}$

Evaluate the determinant of each matrix.

12. $\begin{bmatrix} -1 & 2 & -2 \\ 0 & 1 & 3 \\ 4 & 2 & -1 \end{bmatrix}$ **13.** $\begin{bmatrix} 2 & 1 & 2 \\ -1 & 0 & 5 \\ 0 & 4 & 1 \end{bmatrix}$ **14.** $\begin{bmatrix} 2 & 4 & 3 \\ -3 & 0 & -2 \\ -1 & 3 & 0 \end{bmatrix}$

15. $\begin{bmatrix} 2 & 6 & -1 \\ 1 & 0 & 0 \\ 1 & 3 & -2 \end{bmatrix}$ **16.** $\begin{bmatrix} -4 & 0 & 3 \\ 0 & -2 & 3 \\ -1 & 4 & -2 \end{bmatrix}$ **17.** $\begin{bmatrix} 7 & -1 & 3 \\ 1 & 2 & 6 \\ 4 & 1 & 3 \end{bmatrix}$

Determine whether the matrices are multiplicative inverses.

18. $A = \begin{bmatrix} -2 & 2 & 3 \\ 1 & -1 & 0 \\ 0 & 1 & 4 \end{bmatrix}, B = \begin{bmatrix} -\frac{4}{3} & -\frac{5}{3} & 1 \\ -\frac{4}{3} & -\frac{8}{3} & 1 \\ 1 & \frac{2}{3} & 0 \end{bmatrix}$ **19.** $A = \begin{bmatrix} 2 & -17 & 11 \\ -1 & 11 & -7 \\ 0 & 3 & -2 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 4 & -3 \\ 3 & 6 & -5 \end{bmatrix}$

Unit 5 Day 5: Solving the Systems of Equations

There are two ways to solve a system of equations algebraically:

- Substitution – substitute one equation into the other.
- Elimination – get rid of one variable on both equations and solve.

SUBSTITUTION	ELIMINATION
1. Solve one equation for one-variable 2. substitute that equation into the other equation 3. solve for the variable 4. solve for the remaining variable 5. check answers	1. line up variables 2. eliminate one variable by adding the equations 3. solve resulting equation for the variable 4. solve for the other variable 5. check answers.

Practice: Solve the system using substitution

$\begin{cases} y = x + 3 \\ 5x + y = 9 \end{cases}$	$\begin{cases} 2x + y = -1 \\ 6x - 3y = -33 \end{cases}$	$\begin{cases} 2x - 3y = 6 \\ x + y = -12 \end{cases}$

Practice: Solve the system using elimination

$\begin{cases} 3x + y = -9 \\ -3x - 2y = 12 \end{cases}$	$\begin{cases} 3x + 7y = 15 \\ 5x + 2y = -4 \end{cases}$	$\begin{cases} 4x + 3y = -6 \\ 5x - 6y = -27 \end{cases}$

Solving using Matrices:

The variables make up the variable matrix. The coefficients are the "A" matrix. The constants are the "B" matrix:

Example:

$$\begin{cases} 2x + 3y = 11 \\ x + 2y = 6 \end{cases}$$

Practice:

$$\begin{cases} 5a + 3b = 7 \\ 3a + 2b = 5 \end{cases}$$

Solving Systems with 3 Equations

Elimination: In elimination you will combine 2 of the equations such that you eliminate a variable. Next you combine two other equations to eliminate the same variable. Now you have 2 equations and 2 unknowns, solve just as you did in section with two variables, then find the 3rd variable.

Example: Solve the system

$x + y + z = 6$

$2x - y + 3z = 9$

$-x + 2y + 2z = 9$

Practice:

$$\begin{aligned} x + y + z &= -8 \\ x - y - z &= 6 \\ 2x - 3y + 2z &= -1 \end{aligned}$$

Solving Systems of 3 Equations using your Calculator

Remember:

1. Rewrite the system with the coefficients are the "A" matrix. The constants are the "B" matrix
2. Enter your matrix data into the calculator. Enter the 3x3 data into matrix A and the 3x1 data into matrix B.
3. Now to calculate the inverse hit 2nd **MATRIX** select the matrix you want the inverse for and hit **ENTER**
4. Hit **x^{-1}**.The view screen will show: $[A]^{-1}$
5. With the matrix inverse on the screen hit (times) 2nd **MATRIX** [B] **ENTER**. You will see: $[A]^{-1}[B]$ hit **ENTER** one more time.
6. The resulting matrix will be our answer, the matrix that equals X.

Practice:

1. $\begin{cases} x - y - z = -9 \\ 3x + y + 2z = 12 \\ x - y + 2z = 0 \end{cases}$

2. $\begin{cases} x + y + z = 2 \\ 2x + y = 5 \\ x + 3y - 3z = 14 \end{cases}$

Word Problems

Example 1: John inherited $25,000 and invested part of it in a money market account, part in municipal bonds, and part in a mutual fund. After one year, he received a total of $1,620 in simple interest from the three investments. The money market paid 6% annually, the bonds paid 7% annually, and the mutually fund paid 8% annually. There was $6,000 more invested in the bonds than the mutual funds. Find the amount John invested in each category.

Example 2: Your company has three acid solutions on hand: 30%, 40%, and 80% acid. It can mix all three to come up with a 100-gallons of a 39% acid solution. If it interchanges the amount of 30% solution with the amount of the 80% solution in the first mix, it can create a 100-gallon solution that is 59% acid. How much of the 30%, 40%, and 80% solutions did the company mix to create a 100-gallons of a 39% acid solution?

Example 3: Five hundred tickets were sold for a certain music concert. The tickets for the adults sold for $7.50, the tickets for the children sold for $4.00, and tickets for senior citizen sold for $3.50. The revenue for the Monday performance was $3,025. Twice as many adult tickets were sold as children tickets. How many of each ticket was sold?

Did You Hear About...

A	B	C	D	E	F
G	H	I	J	K	L ?

Solve each system of equations below using multiplication with the addition method. Find the solution in the answer column and notice the word next to it. Write this word in the box containing the letter of that exercise. Keep working and you will hear about some "udder" nonsense.

(A) $5x - 2y = 4$
$3x + y = 9$

(B) $3x - 5y = 13$
$x - 2y = 5$

(C) $7x + 2y = -1$
$3x - 4y = 19$

(D) $x + 2y = 6$
$5x + 3y = 2$

(E) $2x + 3y = 7$
$3x + 4y = 10$

(F) $7x - 3y = -5$
$3x + 2y = 11$

(G) $3x - 5y = 7$
$5x - 2y = -1$

(H) $4x + 3y = 9$
$3x + 4y = 12$

(I) $5x - 3y = 16$
$4x + 5y = -2$

(J) $4x - 3y = -20$
$-x - 8y = 5$

(K) $-3x + 7y = -1$
$-2x + 5y = 0$

(L) $5x + 6y = -11$
$3x + y = -4$

TWEET	(1, 2)
HIS	(2, 1)
SELLING	(−5, 0)
BIRDSEED	(−1, −2)
UDDER	(2, 0)
THE	(2, 3)
SINGING	(−5, 4)
STARTED	(2, −2)
FED	(−2, 4)
BUTTER	(−1, 3)
COWS	(1, 4)
MILK	(−1, −1)
FARMER	(1, −2)
AND	(0, 3)
WINGS	(2, −4)
WHO	(1, −4)
MOO	(1, 3)
CHEEP	(5, 2)
BEEF	(3, −2)

1. $\begin{cases} x + y + z = -1 \\ 2x - y + 2z = -5 \\ -x + 2y - z = 4 \end{cases}$

2. $\begin{cases} x + y + z = 3 \\ 2x - y + 2z = 6 \\ 3x + 2y - z = 13 \end{cases}$

3. $\begin{cases} 2x + y = 9 \\ x - 2z = -3 \\ 2y + 3z = 15 \end{cases}$

4. $\begin{cases} x - y + 2z = 10 \\ -x + y - 2z = 5 \\ 3x - 3y + 6z = -2 \end{cases}$

5. $\begin{cases} 2x - y + z = -4 \\ 3x + y - 2z = 0 \\ 3x - y = -4 \end{cases}$

6. $\begin{cases} 2x - y - z = 4 \\ -x + 2y + z = 1 \\ 3x + y + z = 16 \end{cases}$

7. $\begin{cases} x + 5y + 5z = -10 \\ x + y + z = 2 \\ x + 2y + 3z = -3 \end{cases}$

8. $\begin{cases} x - y - z = 0 \\ x - 2y - 2z = 3 \\ -2x + 2y - z = 3 \end{cases}$

9. $\begin{cases} 3x + y + z = 6 \\ 3x - 2y + 2z = 14 \\ 3x + 3y - 3z = -6 \end{cases}$

10. $\begin{cases} x + y + z = -2 \\ 2x + 2y - 3z = 11 \\ 3x - y + z = 4 \end{cases}$

11. $\begin{cases} x - 5y + z = 3 \\ x + 2y - 2z = -12 \\ 2x + 2z = 6 \end{cases}$

12. $\begin{cases} 2x + 3z = 2 \\ 3x + 6y = 6 \\ x - 2z = 8 \end{cases}$

13. $\begin{cases} x + y - z = 0 \\ 3x - y + z = 4 \\ 5x + z = 7 \end{cases}$

14. $\begin{cases} x - 2y = 1 \\ x + 3y + z = 0 \\ 2x - 2z = 18 \end{cases}$

15. $\begin{cases} x + y + 4z = 5 \\ -2x + 2z = 3 \\ 3x + y - 2z = 0 \end{cases}$

16. $\begin{cases} 3x + 2y + 2z = 4 \\ -6x + 4y - 2z = -9 \\ 9x - 2y + 2z = 10 \end{cases}$

17. $\begin{cases} 2x - 3y + z = -3 \\ x - 5y + 7z = -11 \\ -10x + 4y - 6z = 28 \end{cases}$

18. $\begin{cases} x + y + z = -8 \\ x - y - z = 6 \\ 2x - 3y + 2z = -1 \end{cases}$

19. $\begin{cases} 14x - 3y + 5z = -15 \\ 3x + 2y - 6z = 10 \\ 7x - y + 4z = -5 \end{cases}$

20. $\begin{cases} 5x - 3y + 2z = 39 \\ 4x + 4y - 3z = 34 \\ 3x - 2y + 6z = 14 \end{cases}$

21. $\begin{cases} x + y + z = 6 \\ 2x - y + 2z = 6 \\ -x + y + 3z = 10 \end{cases}$

22. $\begin{cases} 2x + y - z = 3 \\ 3x - y + 3z = 3 \\ -x - 3y + 2z = 3 \end{cases}$

23. $\begin{cases} 2x - 3y + z = 4 \\ -2x + 3y - z = -4 \\ 6x - 9y + 3z = 12 \end{cases}$

24. $\begin{cases} x + y - z = 1 \\ x + 2z = 3 \\ 2x + 2y = 4 \end{cases}$

Write and solve a system of equations for each problem.

25. The sum of three numbers is −2. The sum of three times the first number, twice the second number, and the third number is 9. The difference between the second number and half the third number is 10. Find the numbers.

26. Monica has \$1, \$5, and \$10 bills in her wallet that are worth \$96. If she had one more \$1 bill, she would have just as many \$1 bills as \$5 and \$10 bills combined. She has 23 bills total. How many of each denomination does she have?

1. The sum of three numbers is 16. The largest number is equal to the sum of the other two, and 3 times the smallest number is 1 more than the largest. Find the three numbers.

2. The sum of three numbers is 24. Twice the smallest number is 2 less than the largest number, and the largest number is equal to the sum of the other two. What are the three numbers?

3. A cashier has 25 coins consisting of nickels, dimes, and quarters with a value of $4.90. If the number of dimes is 1 less than twice the number of nickels, how many of each type of coin does she have?

4. A theater has tickets at $6 for adults, $3.50 for students, and $2.50 for children under 12 years old. A total of 278 tickets were sold for one showing with a total revenue of $1300. If the number of adult tickets sold was 10 less than twice the number of student tickets, how many of each type of ticket were sold for the showing?

5. The perimeter of a triangle is 19 cm. If the length of the longest side is twice that of the shortest side and 3 cm less than the sum of the lengths of the other two sides, find the lengths of the three sides.

6. The measure of the largest angle of a triangle is 10° more than the sum of the measures of the other two angles and 10° less than 3 times the measure of the smallest angle. Find the measures of the three angles of the triangle.

7. Jovita divides $17,000 into three investments: a savings account paying 6% annual interest, a bond paying 9%, and a money market fund paying 11%. The annual interest from the three accounts is $1540, and she has three times as much invested in the bond as in the savings account. What amount does she have invested in each account?

8. Adrienne has $6000 invested among a savings account paying 3%, a time deposit paying 4%, and a bond paying 8%. She has $1000 less invested in the bond than in her savings account, and she earned a total of $260 in annual interest. What has she invested in each account?

Unit 5 Day 7: Solving Systems with Cramer's Rule and using Gaussian Elimination

Suppose you have a system of equations and you are only interested in the solution of one variable. With the matrix methods learned so far, this is not possible. Cramer's Rule is a theorem in linear algebra, which may be used to solve systems for only the desired variables. This theorem was derived by a Swiss mathematician, Gabriel Cramer, (1704-1752) and uses ratios of determinant values to solve for the variables.

So, given a system of equations: $\begin{cases} 2x + y + z = 3 \\ x - 2y - z = 0 \\ x + 2y + z = 0 \end{cases}$ **Find (x,y,z)**

Convert into a matrix equation:

We can now make several determinants out of this system.

- First, D is the determinant of the coefficient matrix.
- We are next going to make three other matrices to help us calculate the values of our variables. This will be done by substituting the one-column constant matrix into the coefficient matrix. We will make three new matrices;
    - one for the x-column values,
    - one for the y-column values,
    - one for the z-column values.
- D_x is the determinant formed by replacing the x coefficient column values with the constant matrix.
- D_y is the determinant formed when you replace the y-column values with the constant matrix.
- Lastly, when you replace the z-column values with the constant matrix you get the D_z determinant.

Look carefully at the matrices. Notice the substitution made to form the matrices:

D_x= D_y= D_z=

D=

Cramer's rule says that the variable is equal to the ratio of the variable determinant to the coefficient determinant:
$x = \frac{Dx}{D}$ $y = \frac{Dy}{D}$ *and* $z = \frac{Dz}{D}$

Therefore: x = y = z =

Summary:
1) The variable determinants are formed by substituting the constant matrix into the column of the coefficient matrix that houses the coefficients for the given variable.
2) The ratio of the variable determinant and the coefficient determinant equals the value of the variable.

$$x = \frac{Dx}{D} \quad , \quad y = \frac{Dy}{D} \quad \text{and} \quad z = \frac{Dz}{D}$$

3) Now to find for example the x value, all we need to do is solve the determinants D and D_x.
4) We need to remember with a 3x3 matrix the determinant is solved by expanding the matrix by rewriting column 1 and 2, summing the products of the down-to-the-rights and subtracting the sum of the down-to the-left products.

Example: Solve using Cramer's Rule –

$\begin{cases} 2x + y = 8 \\ x - y = -2 \end{cases}$	1. Set up matrix equation 2. Set up determinants and solve 3. Solve the ratios of Dx /D and Dy/D . 4. Check your answers for (x,y)

All systems of 3 equations would be really nice to solve if they were all like:

$$\begin{aligned} x - 2y + z &= -4 \\ -4x + y &= -7 \\ z &= -4 \end{aligned}$$

Gaussian Elimination

Gaussian Elimination (aka row echelon form) is an effective algorithm (a step by step procedure for calculations) that may be used to reduce systems of 3 equations into a triangular shaped form:

$$\begin{aligned} x - 2y + z &= -4 \\ -4x + y &= -7 \\ z &= -4 \end{aligned}$$

In a college level algebra class you would learn how to perform Gaussian elimination to a matrix, in this class we will work with systems of equations. We saw last class that systems are easily converted to matrix equations.

To perform Gaussian Elimination on a system of equations, one uses a sequence of elementary row operations to modify the system until the last row of the system is a variable equal to a number, the second row is 2 variables equal to a number and the 1st row is 3 variables equal to a number.

There are three types of elementary row operations:
1) Swapping two rows,
2) Multiplying a row by a non-zero number, and
3) Adding a multiple of one row to another row.
NOTE: You will use one row to change another without actually changing the one row.
For example:
Below we will add – 4 times row 3 to row 2 so to change row 2.

Solve using Gaussian Elimination:

1. $\begin{cases} 2x - y + 3z = 13 \\ 4x + 3y - 2z = 5 \\ x - y - 4z = -4 \end{cases}$

2. $\begin{cases} x - 2y + 3z = 4 \\ 2x + y - 4z = 3 \\ -3x + 4y - z = -2 \end{cases}$

Unit 5 Day 7 Practice: Solving Systems with Cramer's Rule and using Gaussian Elimination

Use Cramer's Rule to solve each system.

1) $2x - 3y = 10$
$-6x + 2y = -5$

2) $3x + 4y = -6$
$-2x + 3y = -13$

3) $4x - 2y = -8$
$-x + y = -13$

4) $4x - y = 4$
$x + 4y = -16$

5) $-3x - 3y = -15$
$6x + 6y = 0$

6) $x + 6y = 13$
$x + 4y = 9$

7) $6x - 2y = -5$
$6x + 2y = -11$

8) $5x - y = 25$
$x - y = 9$

9) $3x - y = -13$
$6x + 3y = -21$

10) $-5x - 6y = 11$
$-6x - 4y = 18$

11) $x - 5y + 4z = -7$
$2x - 2y + 4z = 10$
$-x - 3y = -5$

12) $4x - 2y + 5z = 6$
$-4y + z = 6$
$2x - 3y + 3z = -4$

13) $5x + y = -8$
$-5x - y + 3z = -5$
$6x + 3y + 3z = -13$

14) $-x + z = -11$
$3x + 5y + 4z = -9$
$-x - 5y + 3z = -23$

15) $-6x + 6y + z = 5$
$-4y + 5z = -13$
$-2x + 2y - 6z = 8$

16) $-3x + y + 6z = -17$
$-4x + 2z = 4$
$-x - y + 5z = -15$

17) $-5y + 3z = -12$
$-3x + 2y - 2z = -11$
$x - 5y + 3z = -9$

18) $4x - y - 3z = -6$
$6x + 5y + 6z = -13$
$-4x + y = 5$

19) $-6x - y + 5z = 26$
$4x + y - 2z = -22$
$5x - 5z = -15$

20) $2x - 5y - 4z = 3$
$-y + z = -5$
$2x + 4y + 5z = -12$

Solve the following Linear Systems of Equations by Gaussian Elimination:

1) $\begin{cases} 4x+2y-6z=34 \\ 2x+y+3z=3 \\ 6x+3y-3z=37 \end{cases}$

2) $\begin{cases} 3x-y+z=-1 \\ 2x+3y+z=4 \\ 5x+4y+2z=5 \end{cases}$

3) $\begin{cases} 2x+y+z=-2 \\ 2x-y+3z=6 \\ 3x-5y+4z=7 \end{cases}$

4) $\begin{cases} 6x+2y-4z=15 \\ -3x-4y+2z=-6 \\ 4x-6y+3z=8 \end{cases}$

5) $\begin{cases} 3x+3z=0 \\ 2x+2y=2 \\ 3y+3z=3 \end{cases}$

6) $\begin{cases} x+3y-3z=12 \\ 3x-y+4z=0 \\ -x+2y-z=1 \end{cases}$

7) $\begin{cases} x+y+z=3 \\ 2x-y-z=0 \\ x+2y-z=-1 \end{cases}$

8) $\begin{cases} 2x-4y+z=10 \\ x+2y-z=1 \\ -x-3y+2z=0 \end{cases}$

9) $\begin{cases} 3x+y-z=4 \\ x+2y+2z=5 \\ 4x+y-z=3 \end{cases}$

10) $\begin{cases} 5x+6y-5z=-1 \\ 3x-4y-3z=7 \\ -2x+5y+z=-4 \end{cases}$

11) $\begin{cases} 2x+3z=1 \\ 3x-5y=10 \\ 4y-3z=13 \end{cases}$

12) $\begin{cases} 3y+4z=6 \\ 3x-5z=3 \\ 2x+5y=2 \end{cases}$

13) $\begin{cases} \frac{1}{2}x+\frac{1}{3}y=4 \\ \frac{1}{2}y-\frac{1}{4}z=1 \\ \frac{1}{4}x+\frac{1}{2}z=5 \end{cases}$

14) $\begin{cases} \frac{1}{3}x-\frac{1}{3}z=-2 \\ \frac{1}{6}y+\frac{1}{3}z=7 \\ \frac{2}{3}x+\frac{1}{4}y=9 \end{cases}$

Unit 5 Matrices Review

1. State the 4 x 4 identity matrix.
2. Explain why a 3 x 2 matric does not have an inverse.
3. True or false:

$$\begin{bmatrix} 6 & 1 \\ 0 & 4 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{6} & -\frac{1}{24} \\ 0 & \frac{1}{4} \end{bmatrix} = I$$

Use the matrices below to answer questions 4-11

$$R = \begin{bmatrix} 9 & 6 & 7 \\ 2 & 5 & 0 \\ 10 & 3 & 11 \end{bmatrix} \quad S = \begin{bmatrix} 5 & -2 & 4 & 3 \\ 0 & 8 & 0 & -1 \end{bmatrix} \quad T = \begin{bmatrix} 8 & -1 & 6 \\ -7 & 0 & 2 \\ 4 & 9 & -5 \end{bmatrix} \quad U = \begin{bmatrix} 2 & x & -2 & 11 \\ -4 & 3 & 5 & 9 \end{bmatrix}$$

4. What are the dimensions of matrix S?

5. Identify the element r_{23}.

6. The value of x is what element?

7. Which matrices have the same dimensions?

8. R + U =

9. -0.5R=

10. U – S =

11. 3R + T =

Perform the indicated operations <u>without a calculator</u>.

12. $\begin{bmatrix} 1 & 3 & 3 \\ -2 & 4 & 4 \end{bmatrix} - 2\begin{bmatrix} 5 & 5 & -2 \\ 3 & 6 & 2 \end{bmatrix}$

13. $\begin{bmatrix} 6 & 1 \\ 0 & 8 \end{bmatrix} \cdot \begin{bmatrix} -4 & 3 \\ 7 & 11 \\ 3 & 1 \end{bmatrix}$

14. $\begin{bmatrix} -2 & 1 \\ 4 & 0 \\ 2 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 5 \end{bmatrix}$

15. $\begin{vmatrix} 1 & 0 & 2 \\ -1 & 0 & 1 \\ -1 & -2 & 0 \end{vmatrix}$

16. $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} + \begin{bmatrix} -1 & -2 & -3 \\ -4 & -5 & -6 \\ -7 & -8 & -9 \end{bmatrix}$

17. If E= $\begin{bmatrix} 2 & -1 & 3 \\ 0 & 4 & 1 \\ -6 & -2 & 4 \end{bmatrix}$, find E^{-1}

18. Solve the following system using the method of your choice (Cramer's or Gaussian). Do not use a calculator.

$$\begin{aligned} z &= -3x + 3y + 11 \\ -3x + 7y - 7z &= 3 \\ -2x + 2y - 6z &= 30 \end{aligned}$$

19. Solve the following systems using the indicated method.

a. Elimination

$$\begin{aligned} -5x - 5y &= 25 \\ -2x - 4y &= 16 \end{aligned}$$

b. Substitution

$$\begin{aligned} x - 3y &= 5 \\ -3x + 6y &= 8 \end{aligned}$$

c. You pick!

$$\begin{aligned} -4x - 6z &= -12 \\ -6x - 4y - 2z &= 6 \\ -x + 2y + z &= 9 \end{aligned}$$

d. Gaussian

$$\begin{aligned} -6x - y + z &= -7 \\ 4z &= -6 \\ 4x - 24y + 24z &= 17 \end{aligned}$$

e. Cramer's

$$\begin{aligned} 3a + b &= -c + 7 \\ a + 3b - c &= 13 \\ b &= 2a - 1 \end{aligned}$$

f. Gaussian or Cramer's

$$\begin{aligned} 13 &= 3x - y \\ 14y - 3x + 2z &= -3 \\ z &= 2x - 4y \end{aligned}$$

20. Solve the following system using your calculator.

$$\begin{cases} 3x + 3y = 19 + z \\ 5x + 4y - 28 = 2z \\ 2(x + y) - 12 = z \end{cases}$$

Solve for the variables

21. $\begin{bmatrix} 4x \\ 6y \end{bmatrix} + \begin{bmatrix} 6y - 6 \\ 6x + 12 \end{bmatrix} = \begin{bmatrix} -14 \\ 6 \end{bmatrix}$

22. $\begin{vmatrix} 3 & -1 \\ 4 & x \end{vmatrix} = 25$

24. $\begin{bmatrix} 1 & -3 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 22 \end{bmatrix}$

23. $\begin{bmatrix} x & 2 \\ -1 & 5 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ y & 4 \end{bmatrix} = \begin{bmatrix} 9 & 2 \\ -3 & 22 \end{bmatrix}$

25. $\begin{bmatrix} 2z - x & 4w + 5 \\ 2y - w & 10 \end{bmatrix} = \begin{bmatrix} y & 17 \\ 15 & 6z \end{bmatrix}$

26. The first number multiplied by 2 is the opposite of the second number. The third number is subtracted from the product of the second number and 3 to get 20. The sum of the first and third numbers is -5. Use a matrix equation to solve for these three numbers.

27. Ugh! The theater department wants to be able to buy an Audrey II for Little Shop of Horrors next year, and to do so, it actually need $20,000 total revenue from the play. They decide to keep it open for a 4th night, at a far cheaper price so they can make it to their goal. Adult tickets will still be twice as much as student tickets. The sales for the 4th night are exactly like the second night. Set up and solve a matrix equation that shows how much should the theater department charge for each type of ticket to hit their target revenue?

28. You make a VHS tape of your three favorite TV shows for your friend: Family Guy, Lost, and One Tree Hill. You can completely fill the tape with 7 episodes. You want include twice as many episodes of Lost as Family Guy. An episode of Family Guy lasts 30 minutes. An episode of Lost and One Tree Hills lasts 60 minutes. Your VCR tape can only hold 360 minutes of recording. How many episodes of each show can you tape?

29. Mrs. Stevens bought 15 tickets to a spaghetti supper and spent a total of $72. She bought adult tickets for $6 each and child tickets $4 each. How many of each kind of ticket did she buy?

30. A florist is making 5 identical bridesmaid bouquets for a wedding. She has $610 to spend (including tax) and wants 24 flowers for each bouquet. Roses cost $6 each, tulips cost $4 each, and lilies cost $3 each. She wants to have twice as many roses as the other 2 flowers combined in each bouquet. How many roses, tulips, and lilies are in each bouquet?